Objective 4.1: Objects in the plane can be transformed, and those transformations can be described and analyzed mathematically

Question 1: What happens to vertices during transformations?

Assessment tier 1: Focus

Observe a change in coordinates of vertices when the shape is transformed

- Given an object and a transformation identify the corresponding vertices

Find the new location of vertices when a shape is transformed

- Given an object and the degree of transformation sketch the transformation and identify corresponding vertices

Given a transformed figure and the degree of transformation find the original position of the figure

- Given a transformation and the degree of transformation sketch the original and identify the corresponding vertices

Utilize a dilation scale factor to transform an object

- Given a object and the scale factor sketch the transformation
- Given the original and the transformation identify the scale factor

Assessment tier 2: Coherence

Given several different transformations develop a rule for determining how vertices shift

- Write a set of rules for the different types of transformations (reflections, dilations, rotations, and translations)
- What makes a transformation a reflection, a rotation, etc.?

Describe the process for transforming objects such that they are perpendicular to the original

- Using geometric shapes and the principles of transformations discuss what makes two objects parallel or perpendicular
- Using the principle of rotation transform an object such that the transformation is perpendicular to the original (90 degree rotation)

Describe the process for transforming objects such that they are symmetrical around an axis

- Describe the rules for creating / transforming objects such that they are symmetrical around a specific line (start with the axes, more advanced students should be encouraged

to use lines of symmetry not focalized around an axis ie horizontal, vertical, or diagonal lines of symmetry)

- Identify when other kinds of transformations such as rotations can create symmetrical objects (180 degree transformations)

Assessment tier 3: Rigor

Describe the relationship between rotation and reflection as a method of transformation

- When do the two methods of transformation create similar or identical objects? As an extension ask the same question for the other methods of transformation

Discuss how principles of transformation are used in modern and historical methods of construction (bridges, gothic architecture versus romanic)

- Analyze historical objects for lines of symmetry that aid in structural integrity
- What aspects of bridge construction need to utilize the principles of transformation such that the bridge exhibits the greatest strength and stability.
- Discuss what objects and transformations create the most structurally sound buildings

Analyze works of art to determine how principles of transformation can create beauty.

- Introduce how art is critiqued and have students critique art through a mathematical lens (specifically finding symmetry in a work of art).
- Using the principles of transformation have students create their own art to be judged by a separate peer group

Essential Question 2: How would the idea of congruency be used outside of mathematics?

There is no tier 1 assessment with this essential question as it pertains to concepts external to mathematics

Tier 2 - Coherence

Describe a situation in another subject where two concepts or events are congruent

- Looking at the other disciplines examine for times or situations when two things are congruent. When two instruments are playing the same notes but produce different sounds, when two philosophies seek to make the same argument although they may be separated by space or time
- Develop a working definition for what makes something congruent to something else

Examine other disciplines for instances when congruency are of importance / utilized heavily

- Look at the arts, physics, and other disciplines to establish that congruency is something common to the human experience

  When is congruence discouraged in other disciplines?

  - When are congruent shapes discouraged in art and why are they discouraged?

Tier 3 - Rigor

Congruent objects in mathematics follow strict rules. Determine a set of rules for when two things outside of mathematics are congruent

- Does having a similar mode of argument but a different outcome make two ideas congruent?
- Do musical instruments create congruent sounds even though their sounds are created differently?
- Can two communities be congruent to each other in ecology?

Examine aspects of construction for congruency.

- Are there any processes that encourage or require congruency?
- Examine steps of building construction. Are there are steps that are identical between projects? What steps do every project have in common?

How can an artist use congruent objects to create beauty? What about congruent or symmetrical objects makes something beautiful from a baroque perspective?

Essential question 3 - What does it mean for two things to be the same? Are there different degrees of "sameness?"

Tier 1 – Focus

What are the differences between similar and congruent objects?

- Similar objects have identical corresponding angles while congruent objects have identical corresponding angles and sides. How does this affect the degree of "sameness" that two objects might share?

How does a transformation alter the nature of "sameness"?

- Which transformations change the nature of "sameness" and how do they go about doing it?
- If an object is transformed are the two objects required to be similar?

Given two objects in coordinate space determine if they are congruent, similar, or different from each other

Given an object in coordinate space and whether it is congruent, similar, or different to another object sketch the second object

Tier 2 – Coherence

Are two objects of the same chemical composition always exactly the same?

- When does the arrangement of molecules or the method that creates object alter the level of "sameness"?

What are some ways to distinguish identical and similar concepts / objects in other disciplines?

- What distinguishes different kinds of art? Are two works of art made with the same materials but presented in different ways identical? What level of "sameness" do they exhibit?
- How do you determine how similar two cultures are?

Are infinitely long lines similar to other infinitely long lines?

- How is similarity classified when the objects in question cannot be measured?

Tier 3 – Rigor

Create a system for analyzing how similar two items are in a different discipline.

- How similar are two authors writing the same thing about the same event when they are writing in different languages? How do you determine how similar their work is or if it is exactly the same?
- Use scales such as the Mohs scale of mineral hardness as a template for creating other scales that link mineral or other trait similarities. What can this tell you about the level of "sameness" two things exhibit?

Why is it helpful to establish levels of "sameness" between two things?

- This question serves to ask about why we seek to create connections between two objects even if they aren't exactly the same. How does this help us analyze the universe in which we live?

Why is it harmful to establish levels of "sameness" between two things?

- Stereotyping can be very helpful for things like chairs (I assume a chair can support my weight) or a racing car (if I stand in front of it I will get hurt) but when does stereotyping turn into a bad thing? How can you avert this kind of thinking while enforcing the positive aspects of establishing similarity?

Essential Question 4 - What makes a good definition of a shape?

Tier 1 - Focus

Define a shape given its sides and angles

- Use predefined notions of shapes such as ‘triangles have 3 sides’ or ‘obtuse triangles have one angle over 90 degrees’ to define a polygon

Sketch an object given pertinent information about its sides / angles

Examine the principles of transformation to determine if an object’s properties alter after being transformed

- Encourage students to recognize what properties can and cannot be changed during transformation

Compare two objects of the same type and determine if one is a transformation of the other

- Do the corresponding sides and angles equal each other? How do you determine what the corresponding angles and sides are?

Tier 2 – Coherence

How are definitions used in other disciplines to create meaning?

* How do the definitions of different kinds of plants create meaning in biology and how does that meaning influence our interaction with the discipline?

What makes a definition of a word good? Cluttered? Inaccurate?

How do definitions from algebra effect or aid understanding of geometric objects?

* parallelograms require that the opposing sides be parallel. In algebra terms this requires the sides to have identical slope. How does this effect or aid in our understanding of geometric objects?

Tier 3 – Rigor

Create a new 2- or 3-dimensional object and provide a succinct and accurate definition of it such that the new object cannot be confused for another object.

- What are the properties of the object?
- Can students explicitly define an object without unnecessary clutter?
- What makes a definition inaccurate?

What is a situation in which two people might disagree about a definition? Why is this possible?

- Some concepts such as morality, politics, or philosophy do not lend themselves to hard definitions. Two individuals can completely disagree about a concept but still both be correct.

Why do people attempt to define the world in which we live? What good can this lead to? What bad can it lead to?

How many differences need to exist between two concepts in order to differentiate them? Do minor differences make two things separate?

Essential question 5: How is slope used to determine the relationships between lines?

Tier 1 – Focus

How do you determine if two lines are parallel?

How do you determine if two lines are perpendicular?

If a line has twice the slope of another line does that make them similar?

Tier 2 – Coherence

How can the slopes of sides of an object be linked to systems of equations and their solutions?

- The sides of an object intersect at the vertex, which would be the solution to a system of two equations where each side is one part of the system.

What is the relationship between a line and its derivative?

How can slope be used to determine the relationship between trigonometric functions?

Tier 3 – Rigor

What processes require parallel lines to function? Why do they require parallel lines?

- Things like railroads require parallel lines in order to keep the train on the track.

What relationships in lines lend themselves easily to methods of construction? Why do they lend themselves to those methods?

Why isn't the slope of a 3-dimensional object conventionally defined as slope in 1- and 2-dimensions?

Essential question 6: How do you set up a proof?

Tier 1 – Focus

How are the hypothesis and conclusion structured in a proof?

How is a flow-proof written?

How is a paragraph-proof written?

How is a column-proof written?

How does one write an indirect proof?

What is the converse, contrapositive, and inverse or a statement?

What is the difference between a postulate and a proof?

Tier 2 – Coherence

What is the purpose of proof writing and how does that apply to other disciplines like biology or English?

- All disciplines rely on some method of proving an idea or point. Whether evidence based observations from English or more fact based for the sciences.

How does proof writing relate to rhetoric and logic?

- What other methods of argumentation can a student use to prove something (ethos, pathos)? Can those methods apply in mathematics?

What kind of proofs can use inductive reasoning and how would they be constructed?

Tier 3 – Rigor

How do proofs help us analyze the world in which we live?

How would you go about writing a proof about the orbit of objects in our solar system?

How might proof writing help justify methods of building construction or destruction?

Essential question 7: How do you perform a transformation (reflections, translations, and rotations)?

Tier 1 – Focus

How do translations effect the position of an object?

What must be done to an object to rotate it? To rotate it about a point?

How do reflections and rotations relate to one another?

How do you scale an object?

Tier 2 – Coherence

How can you determine if one line is a transformation of another line?

- How would the slope of a transformed line relate to the original?

How would a transformed line relate to another if they are placed in a system of equations?

- Translated lines would result in an inconsistent system while rotated and reflected lines would allow the system to be independent.

How can a transformation be represented as a function of x or y?

Tier 3 - Rigor

What are AC and DC power and how does someone transform one kind of power into another?

In what kinds of art could someone expect to find transformations? What kinds of transformations lend themselves to art?

- More modern art styles have a fascination with several types of transformations and can display them with great acuity. Asking students to analyze art that features different kinds of transformations can help to expose students to what transformations may look like when taken to an extreme.

How can a translated line save computer processing power when used in programming?
* in trigonometric functions a computer can determine what one wavelength would be and then translate any subsequent distance to avoid extra computations.

Essential Question 8 - What tools do you need to make accurate geometric constructions and how do you go about using them?

Tier 1 – Focus

What are the following tools and how are they utilized: protractor, compass, and straight edge?
Why are protractors necessary for drawing accurate geometry constructs?
What kinds of drawings are compasses able to aid in?
What other items can you use to approximate a protractor, compass, or straight edge?
How can these tools help in carrying out transformations?

Tier 2 – Coherence

What tools are necessary for accurate measurement in other disciplines? Why are they needed?
Why is “about 90 degrees” not a good measurement?
What are other tools that are/were used throughout history that help us circumvent issues in mathematics?

Tier 3 – Rigor

Why are protractors and compasses of such particular use in geometry?
* Many tools are used specifically the save time, especially in situations when the event the tool is helping us carry out is not central to a problem.
How can someone avoid using a protractor but still ensure that lines are perpendicular?
How would someone go about inventing a tool for math? What would they need to keep in mind and how would they determine if their tool is a success?
* This concept relates to why we make tools and what makes a particular tool worthwhile. Students should be encouraged to create a physical copy of any tool that they can envision and see where / how it can help with a problem.

Essential question 9 – How do you represent translations and reflections as functions?

Tier 1 – Focus

Given an object and its translation determine the function that transforms the object

Given the translation and the function sketch the original

Given an object and its reflection determine the function that transforms the object

Given the reflection and the function sketch the original

Tier 2 – Coherence

How is the line of symmetry represented in a function when it is not an axis?

- How are vertical / horizontal lies written in slope-intercept form?

When is it appropriate to think of x as a function of y when performing transformations?

- Why would it make a transformation easier to think of x as a function of y?

How would a reflection of a line be carried out using a function?

Tier 3 – Rigor

Why can transformations be represented as functions?

How would the process alter for transformations of 3-dimensional objects?

- What would the line of symmetry be replaced by when discussing 3-dimensions? How would that be represented as a function?

How do parent functions help to illustrate concepts of transformations?