

## SIMILAR CIRCLES

### ALL CIRCLES ARE SIMILAR

In general, two figures are similar if there is a set of transformations that will move one figure exactly covering the other. To prove any two circles are similar, only a translation (slide) and dilation (enlargement or reduction) are necessary. This can always be done by using the differences in the center coordinates to determine the translation and determining the quotient of the radii for the dilation.

#### Example 1

Show that circle C with center  $(-1, 2)$  and radius 3 is similar to circle D with center  $(3, 4)$  radius 5.

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center 4 units to the right and two units up. To enlarge circle C to the same radius as D, the enlargement ratio is the quotient of the radii:  $\frac{5}{3}$ .

#### Example 2

Show that circle C with center  $(0, 2)$  and radius 6 is similar to circle D with center  $(0, -6)$  radius 2.

To transform circle C to the larger circle D we only need to find the translation for the center and the enlargement ratio for the radius. The translation is to slide the center eight units down. To reduce circle C to the same radius as D, the reduction ratio is the quotient of the radii:  $\frac{2}{6} = \frac{1}{3}$ .

## Problems

Show the two given circles are similar by stating the necessary transformations from C to D.

1. C: center (2, 3) radius 5; D: center (-1, 4) radius 10.
2. C: center (0, -3) radius 2; D: center (-2, 5) radius 6.
3. C: center (-2, 8) radius 4; D: center (0, 4) radius 9.
4. C: center (2, 8) radius 5; D: center (-2, 4) radius 1.
5. C: center (12, 32) radius 15; D: center (-1, 4) radius 10.
6. C: center (2, 0) radius 7; D: center (-1, 0) radius 4.

For classes familiar with the equation of a circle.

7. C:  $(x+3)^2 + (y-2)^2 = 9$ ; D:  $(x-1)^2 + (y-5)^2 = 25$
8. C:  $(x-3)^2 + (y-4)^2 = 49$ ; D:  $(x+1)^2 + y^2 = 1$
9. C:  $(x+5)^2 + (y-7)^2 = 9$ ; D:  $(x+1)^2 + (y+1)^2 = 5$

## Answers (translation; dilation)

- |  |                                  |
|--|----------------------------------|
| 1. left 3, up 1; 2                                 | 2. left 2, up 8; 3               |
| 3. right 2, down 4; $\frac{9}{4}$                  | 4. left 4, down 4; $\frac{1}{5}$ |
| 5. left 13, down 28; $\frac{10}{15} = \frac{2}{3}$ | 6. left 3; $\frac{4}{7}$         |
| 7. right 2, up 3; $\frac{5}{3}$                    | 8. left 4, down 4; $\frac{1}{7}$ |
| 9. right 4, down 8; $\frac{\sqrt{5}}{3}$           |                                  |