

NAEP Released Items Aligned to the Iowa Core: Geometry

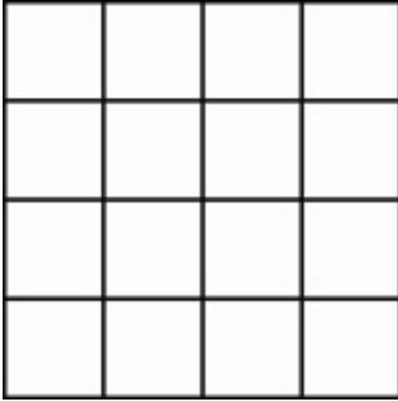
Congruence

G-CO

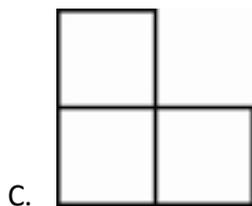
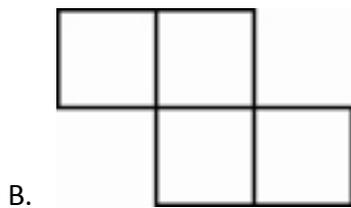
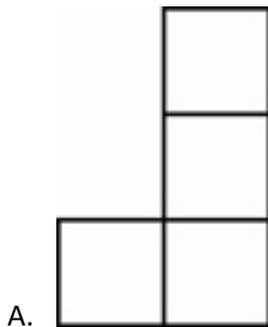
Experiment with transformations in the plane

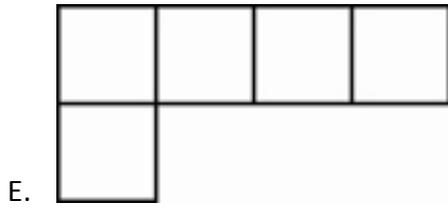
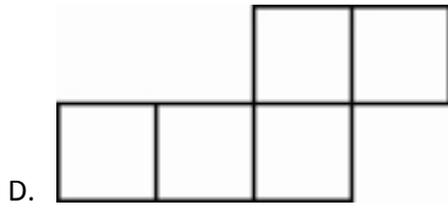
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. (G-CO.1.)
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). (G-CO.2.)
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. (G-CO.3.)
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. (G-CO.4.)

5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. (G-CO.5.)



Identical puzzle pieces have been put together to form the large square shown above. Which of the following could be the shape of each puzzle piece?





2009-8-10-15

Source: National Assessment of Educational Progress, 2011, Grade 8 Mathematics Assessment.

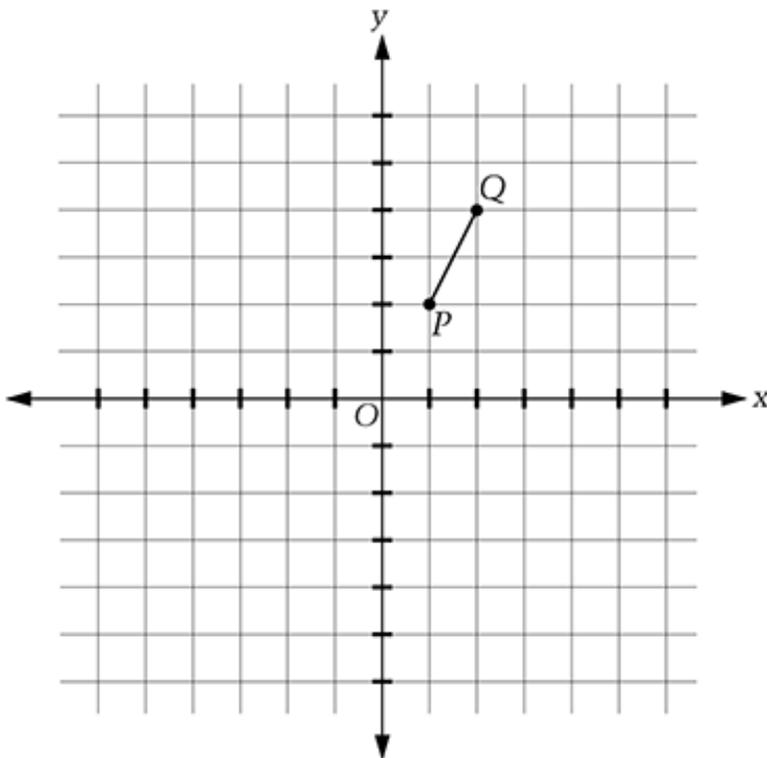
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Perform the following two transformations on the graph below. After each transformation, draw the resulting image of segment PQ.



(1) Rotate the segment  $90^\circ$  counterclockwise about point P. Label the resulting segment with a 1.

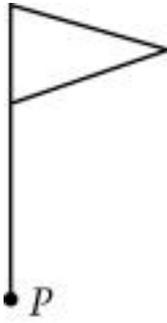
(2) Reflect the segment you drew in part (1) across the x-axis. Label the resulting segment with a 2.



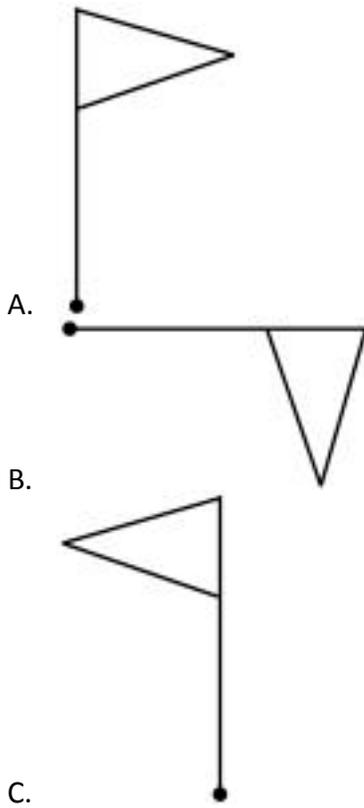
2011-12-2-2

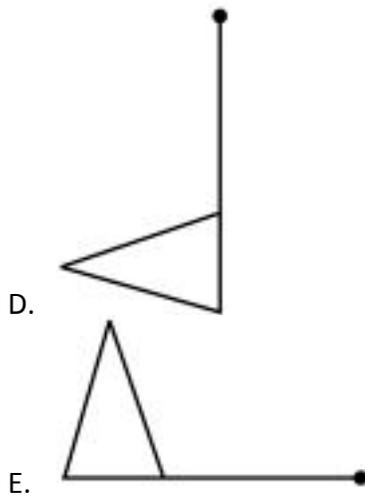
Source: National Assessment of Educational Progress, 2011, Grade 12 Mathematics Assessment.

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The figure above will be rotated clockwise  $180^\circ$  about point  $P$ . Which of the following will be its position after the rotation?





2005-12-12-7

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

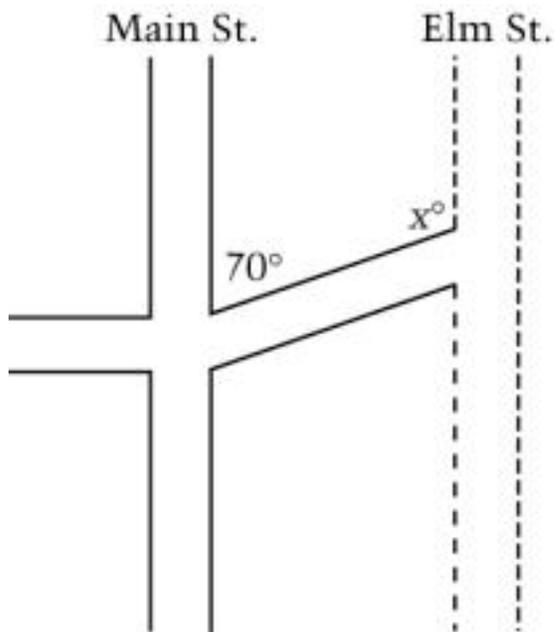
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#### Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. (G-CO.6.)
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. (G-CO.7.)
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. (G-CO.8.)

Prove geometric theorems

9. Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. (G-CO.9.)



In the figure above, Elm Street is to be constructed parallel to Main Street. What is the value of  $x$ ?

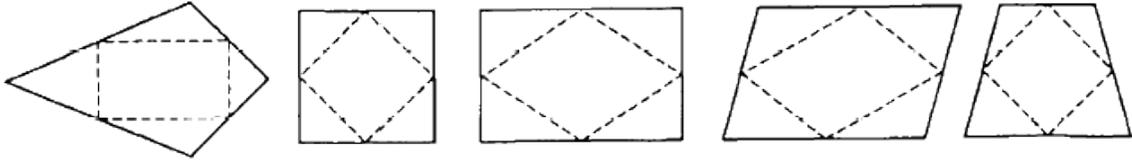
- A. 70
- B. 110
- C. 120
- D. 130
- E. 140

2005-12-12-4

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

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10. Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to  $180^\circ$ ; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. (G-CO.10.)



In the five quadrilaterals shown above, the midpoints of the sides have been joined by broken line segments. Which best describes the five dotted figures formed?

- A. All are parallelograms.
- B. All are rectangles.
- C. All are squares.
- D. All are rhombuses.
- E. No generalization can be made.

1992-12-15-6

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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11. Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. (G-CO.11.)

In the space below, use your ruler to draw a parallelogram that has perpendicular diagonals. Show the diagonals in your sketch.

1996-12-13-2

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

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Which of the following is NOT a property of every rectangle?

- A. The opposite sides are equal in length.
- B. The opposite sides are parallel.
- C. All angles are equal in measure.
- D. All sides are equal in length.
- E. The diagonals are equal in length.

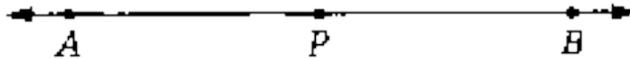
1992-12-5-5

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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### Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. (G-CO.12.)



Which of the following constructions at point P in the figure above will produce an angle of  $45^\circ$ ?

- A. Constructing only the bisector of  $\angle APB$
- B. Constructing only a circle with center at P
- C. Constructing one perpendicular line only
- D. Constructing a circle with center at P and a perpendicular line
- E. Constructing a perpendicular line and an angle bisector

1992-8-15-12

Source: National Assessment of Educational Progress, 1992, Grade 8 Mathematics Assessment.

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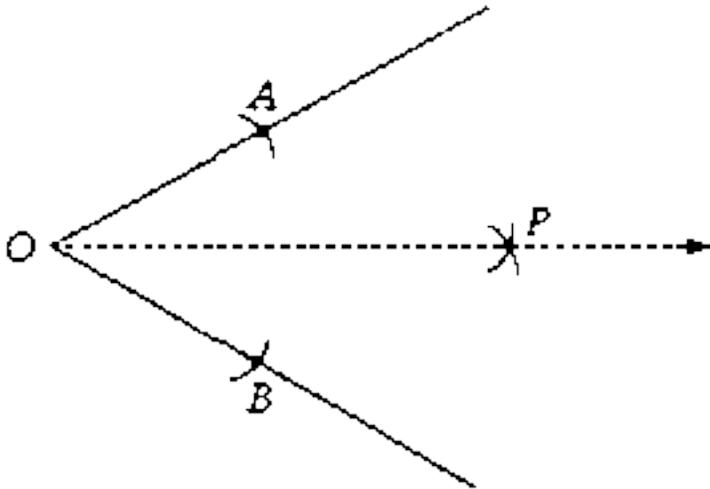
This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.

Describe a procedure for locating the point that is the center of a circular paper disk. Use geometric definitions, properties, or principles to explain why your procedure is correct.

1996-12-10-10

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

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The figure above shows the construction of the angle bisector of  $\angle AOB$  using a compass. Which of the following statements must always be true in the construction of the angle bisector?

Fill in one oval to indicate YES or NO for each statement.

	Yes	No
(a) $OA = OB$	<input type="radio"/>	<input type="radio"/>
(b) $AP = BP$	<input type="radio"/>	<input type="radio"/>
(c) $AB = BP$	<input type="radio"/>	<input type="radio"/>
(d) $OB = BP$	<input type="radio"/>	<input type="radio"/>

1992-12-15-4

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.  
(G-CO.13.)

Similarity, Right Triangles, and Trigonometry

G-SRT

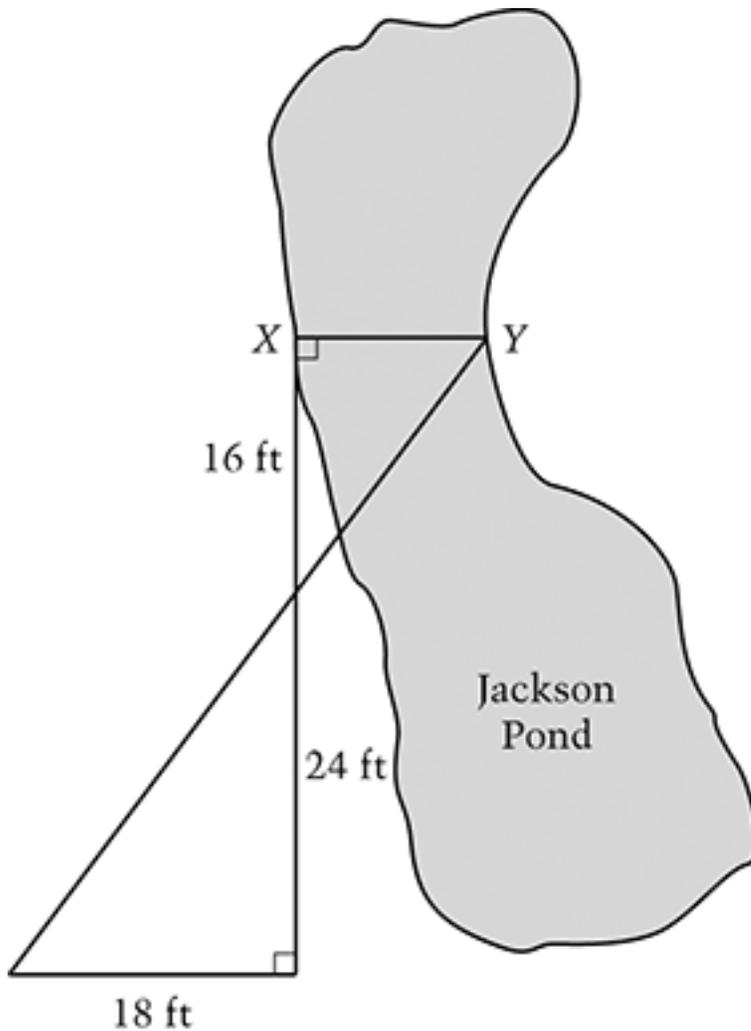
Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:

- a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. (G-SRT.1.)

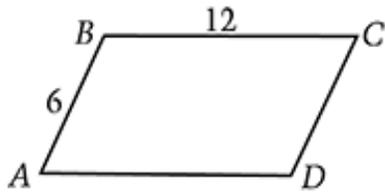
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. (G-SRT.2.)

The figure below shows Jackson Pond.



What is the distance across Jackson Pond from point X to point Y?

- A. 8 feet
- B. 10 feet
- C. 12 feet
- D. 14 feet
- E. 22 feet



Parallelograms ABCD and PQRS above are similar. What is the length of side QR?

- A. 4.5
- B. 9
- C. 12
- D. 15
- E. 18

2011-8-8-14

Source: National Assessment of Educational Progress, 2011, Grade 8 Mathematics Assessment.

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Which of the following pairs of geometric figures must be similar to each other?

- A. Two equilateral triangles
- B. Two isosceles triangles
- C. Two right triangles
- D. Two rectangles
- E. Two parallelograms

2005-12-12-1

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

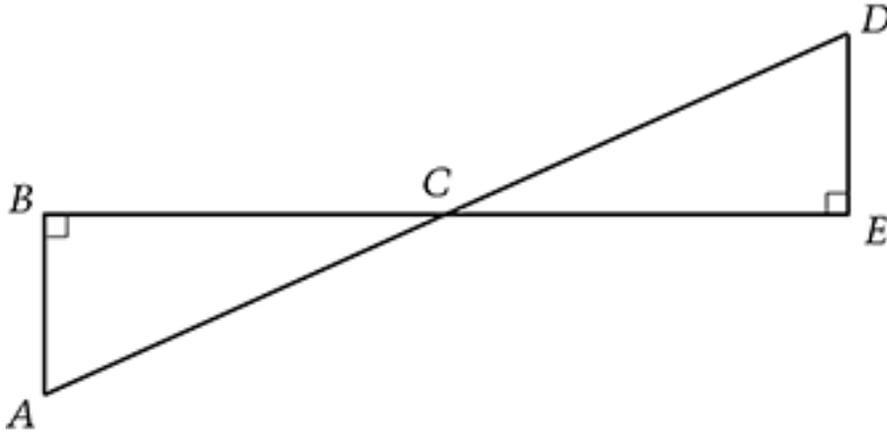
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3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. (G-SRT.3.)

Prove theorems involving similarity

4. Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. (G-SRT.4.)

5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. (G-SRT.5.)



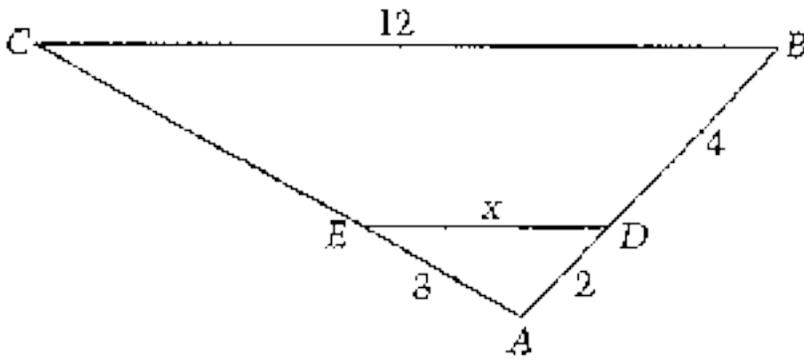
Given:  $C$  is the midpoint of  $\overline{BE}$ .  
 $\angle B$  and  $\angle E$  are right angles.

Prove that  $\overline{AC} \cong \overline{DC}$  and give a reason for each statement in your proof.

2009-12-7-7

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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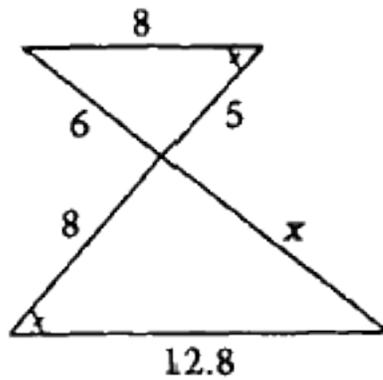
If triangles  $ADE$  and  $ABC$  shown in the figure above are similar, what is the value of  $x$ ?

- A. 4
- B. 5
- C. 6
- D. 8
- E. 10

1996-12-12-1

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

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In the figure above, the two triangles are similar. What is the value of  $x$ ?

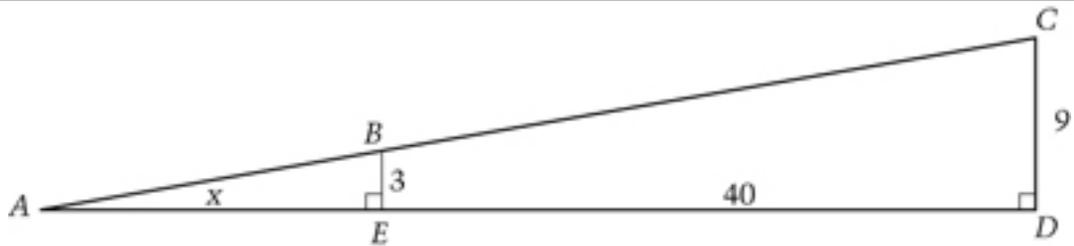
1992-12-14-9

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. (G-SRT.6.)



The figure above shows two right angles. The length of  $AE$  is  $x$  and the length of  $DE$  is 40.

Show all of the steps that lead to finding the value of  $x$ . Your last step should give the value of  $x$ .

2007-8-7-16

Source: National Assessment of Educational Progress, 2007, Grade 8 Mathematics Assessment.

7. Explain and use the relationship between the sine and cosine of complementary angles. (G-SRT.7.)

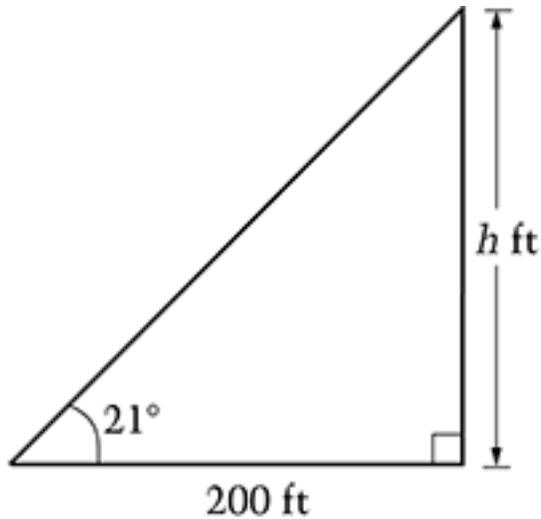
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. (G-SRT.8.)

On level ground from a distance of 200 feet, the angle of elevation to the top of a building is  $21^\circ$ , as shown in the figure above. What is the height  $h$  of the building, to the nearest foot?

- A. 72
- B. 77
- C. 187
- D. 201
- E. 521

2011-12-2-6

Source: National Assessment of Educational Progress, 2011, Grade 12 Mathematics Assessment.



**Note:** Figure not drawn to scale.

On level ground from a distance of 200 feet, the angle of elevation to the top of a building is  $21^\circ$ , as shown in the figure above. What is the height  $h$  of the building, to the nearest foot?

- A. 72
- B. 77
- C. 187
- D. 201
- E. 521

2009-12-7-11

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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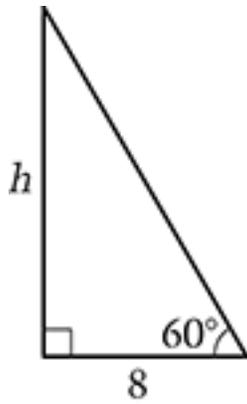
A cat lies crouched on level ground 50 feet away from the base of a tree. The cat can see a bird's nest directly above the base of the tree. The angle of elevation from the cat to the bird's nest is  $40^\circ$ . To the nearest foot, how far above the base of the tree is the bird's nest?

- A. 32
- B. 38
- C. 42
- D. 60
- E. 65

2005-12-12-15

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

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What is the value of  $h$  in the figure above?

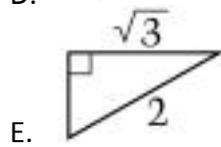
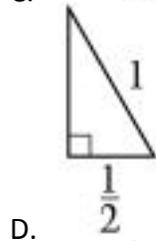
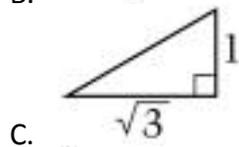
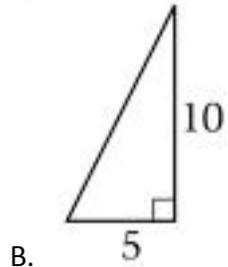
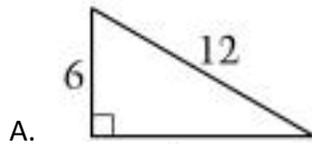
- A.  $4\sqrt{3}$
- B.  $8\sqrt{2}$
- C.  $8\sqrt{3}$
- D.  $12\sqrt{2}$
- E.  $12\sqrt{3}$

2011-12-2-2

Source: National Assessment of Educational Progress, 2011, Grade 12 Mathematics Assessment.

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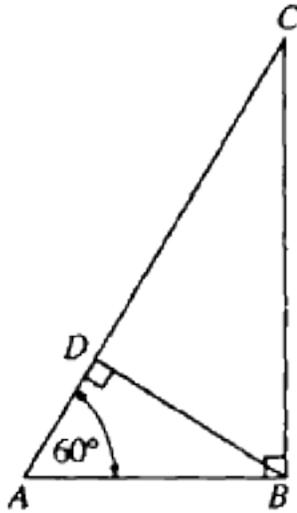
Which of the right triangles below could NOT be a  $30^\circ - 60^\circ - 90^\circ$  triangle?



2005-12-4-13

Source: National Assessment of Educational Progress, 2005, Grade 12 Mathematics Assessment.

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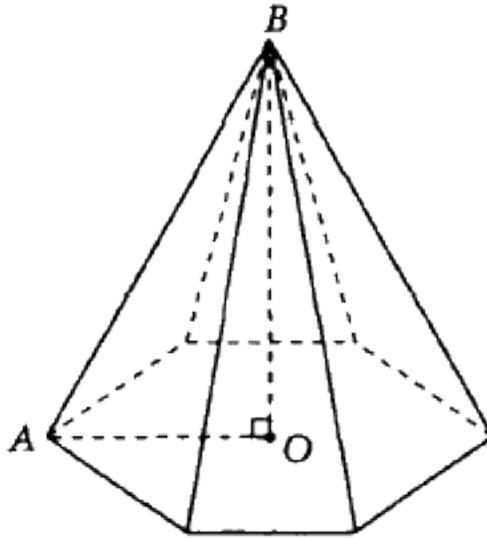
In  $\triangle ABC$  shown above,  $AC = 12$ . What is the length of segment  $BD$ ?

- A.  $3\sqrt{2}$
- B.  $3\sqrt{3}$
- C. 6
- D.  $6\sqrt{2}$
- E.  $6\sqrt{3}$

1992-12-5-16

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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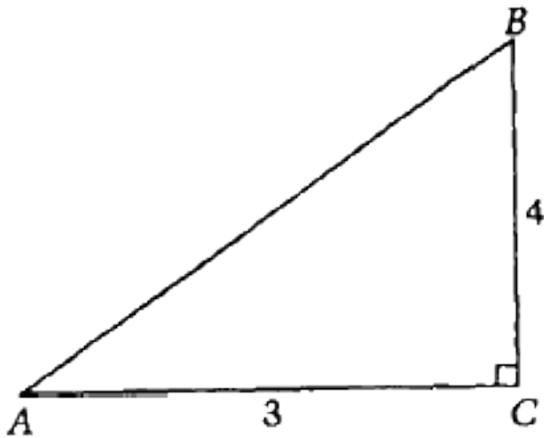


The base of the pyramid shown above is a regular hexagon with side of length 12. If point  $O$  is the center of the base and the length of  $OB$  is 15, what is the degree measure of angle  $OAB$  to the nearest whole number?

1992-12-12-8

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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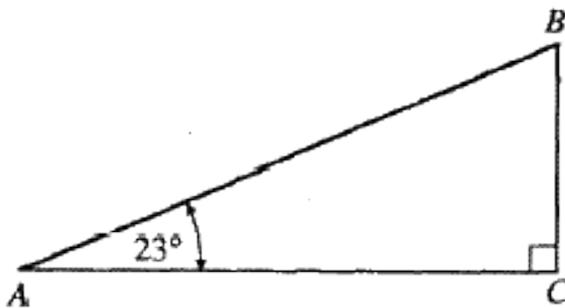
In right triangle  $ABC$  above,  $\cos A =$

- A.  $3/5$
- B.  $3/4$
- C.  $4/5$
- D.  $4/3$
- E.  $5/3$

1992-12-15-10

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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In right triangle  $ABC$  shown above,  $AB = 14$ . What is  $BC$  to the nearest hundredth?

1990-12-9-18

Source: National Assessment of Educational Progress, 1990, Grade 12 Mathematics Assessment.

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Apply trigonometry to general triangles

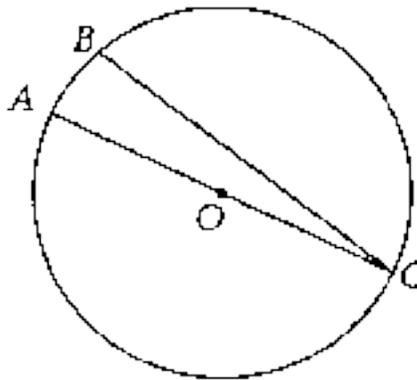
9. (+) Derive the formula  $A = \frac{1}{2} ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (G-SRT.9.)
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems. (G-SRT.10.)
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (G-SRT.11.)

Circles

G-C

Understand and apply theorems about circles

1. Prove that all circles are similar. (G-C.1.)
2. Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. (G-C.2.)



Point  $O$  is the center of the circle above. Line segment  $AC$  is a diameter of the circle. Line segment  $BC$  does not pass through the center of the circle. Which of the following is true?

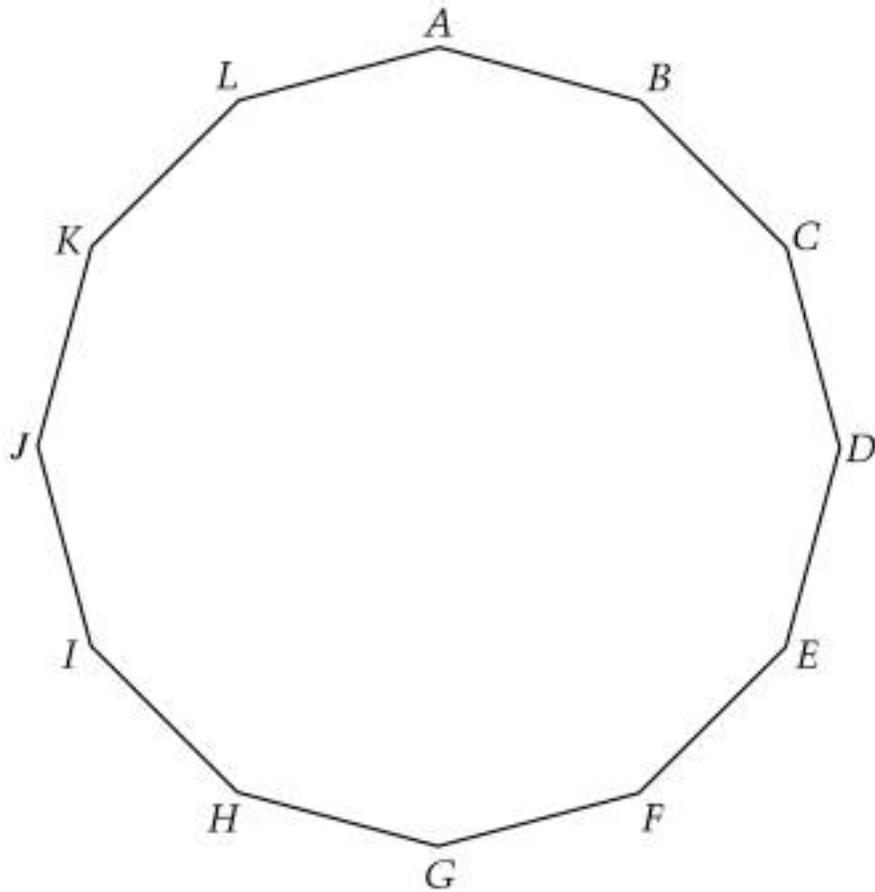
- F.  $AC$  is longer than  $BC$ .
- G.  $BC$  is longer than  $AC$ .
- H.  $AC$  and  $BC$  are the same length.
- I.  $BC$  is twice as long as  $OA$ .
- J. The lengths of  $AC$  and  $BC$  change, depending on how this piece of paper is turned.

1992-8-15-16

Source: National Assessment of Educational Progress, 1992, Grade 8 Mathematics Assessment.

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The following question refers to the following information.

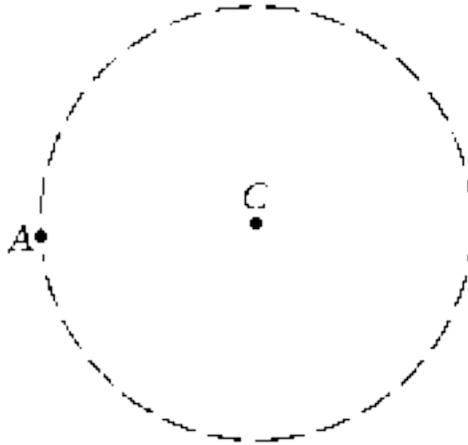


Each of the 12 sides of the figure above has the same length.

Which of the following angles has a measure of  $90^\circ$  ?

- A. Angle  $ABI$
- B. Angle  $ACG$
- C. Angle  $ADF$
- D. Angle  $ADJ$
- E. Angle  $AEH$

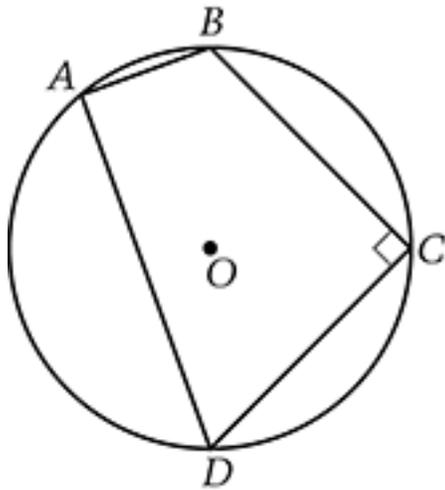
On the circle with center  $C$  shown below, use a protractor to locate and label a point  $B$  that creates an arc  $AB$  with measure  $235^\circ$ . Darken this arc.



1996-12-13-7

Source: National Assessment of Educational Progress, 1996, Grade 12 Mathematics Assessment.

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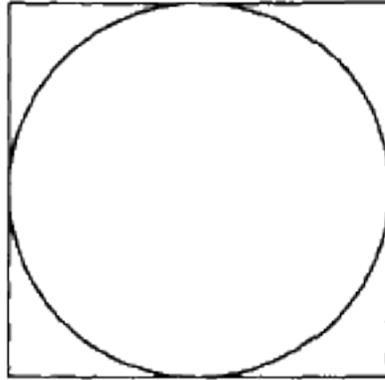
Quadrilateral ABCD is inscribed in circle O , and  $\angle C$  is a right angle, as shown above. Segment AB is not parallel to segment DC . Which of the following statements must be true?

- A.  $\angle A \cong \angle B$
- B.  $\angle B \cong \angle D$
- C.  $\angle B$  is a right angle.
- D.  $\overline{AC}$  is a diameter of circle O .
- E.  $\overline{BD}$  is a diameter of circle O .

2009-12-7-4

Source: National Assessment of Educational Progress, 2009, Grade 12 Mathematics Assessment.

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The length of a side of the square above is 6. What is the length of the radius of the circle?

- A. 2
- B. 3
- C. 4
- D. 6
- E. 8

1992-12-15-2

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. (G-C.3.)

4. (+) Construct a tangent line from a point outside a given circle to the circle. (G-C.4.)

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. (G-C.5.)

Expressing Geometric Properties with Equations

G-GPE

Translate between the geometric description and the equation for a conic section

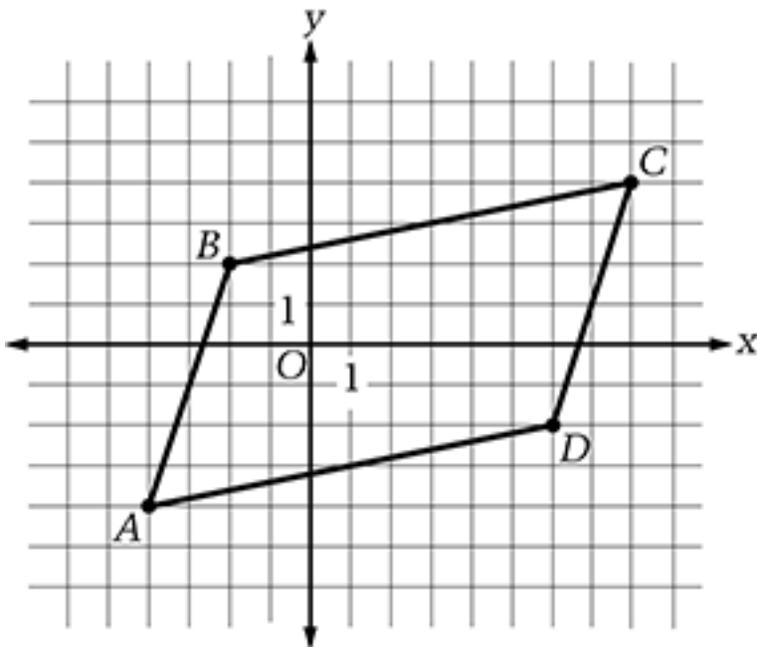
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. (G-GPE.1.)

2. Derive the equation of a parabola given a focus and directrix. (G-GPE.2.)

3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. (G-GPE.3.)

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point  $(1, \sqrt{3})$  lies on the circle centered at the origin and containing the point  $(0, 2)$ . (G-GPE.4.)



In the figure above, the vertices of ABCD are  $A(-4, -4)$ ,  $B(-2, 2)$ ,  $C(8, 4)$  and  $D(6, -2)$ .

Give a mathematical justification that ABCD is a parallelogram.

2011-12-2-12

Source: National Assessment of Educational Progress, 2011, Grade 12 Mathematics Assessment.

5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). (G-GPE.5.)
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. (G-GPE.6.)

7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. (G-GPE.7.)

The endpoints of a line segment are the points with coordinates (2, 1) and (8, 9). What are the coordinates of the midpoint of this line segment?

- A. (2,  $3\frac{1}{2}$ )
- B. (3, 4)
- C. (5, 5)
- D. ( $4\frac{1}{2}$ ,  $5\frac{1}{2}$ )
- E. (10, 10)

2005-8-12-9

Source: National Assessment of Educational Progress, 2005, Grade 8 Mathematics Assessment.

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What is the distance between the points (2, 10) and (-4, 2) in the  $xy$ -plane?

- A. 6
- B. 8
- C. 10
- D. 14
- E. 18

1992-12-7-7

Source: National Assessment of Educational Progress, 1992, Grade 12 Mathematics Assessment.

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Geometric Measurement and Dimension

G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. (G-GMD.1.)
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. (G-GMD.2.)
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. (G-GMD.3.)

Visualize relationships between two-dimensional and three-dimensional objects

IA.7.Plot points in three-dimensions.



(IA) Use diagrams consisting of vertices and edges (vertex-edge graphs) to model and solve problems related to networks.

IA.8. Understand, analyze, evaluate, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.

IA.9. Model and solve problems using at least two of the following fundamental graph topics and models: Euler paths and circuits, Hamilton paths and circuits, the traveling salesman problem (TSP), minimum spanning trees, critical paths, vertex coloring.

IA.10. Compare and contrast vertex-edge graph topics and models in terms of:

- properties
- algorithms
- optimization
- types of problems that can be solved