Concepts of Probability

Topics:

- Experiments, outcomes, sample space, and events
- Union, Intersection, complement, disjoint Events
- Probability
- Axioms of Probability
- Properties of Probability

Experiments, outcomes, sample space, and events

Experiment	Possible **Outcomes**
Toss a dice	1, 2, 3, 4, 5, 6
Flip a coin	H, T
Flip 2 coins	HH, HT, TH, TT
Examine 2 fuses in sequence (fail or pass)	PP, PF, FP, FF

The **sample space**, *S*, of an experiment is the set (collection) of all possible outcomes from an experiment

- An **event**, *A*, is a subset of the sample space S.

Ex. Three fuses are examined in sequences and each receive a pass (P) or fail (F) rating as a result of the inspection.

(1) S = sample space = {PPP, PPF, FPP, PFP, PFF, FPF, FFP, FFF}

(2) Let *A* denote the event that exactly one fuse fails inspection. How would *A* be defined?

A = {PPF, FPP, PFP}

Union, Intersection, complement, disjoint events

- Consider the fuses example: let B denote the event that at most one fuse fails inspection. What is $A \cup B$? $A \cap B$? A'? *B'?* Are events A and B disjoint?

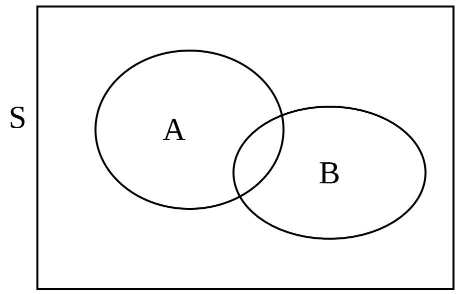
A = {PPF, FPP, PFP}

B = { PPP, PPF, FPP, PFP}

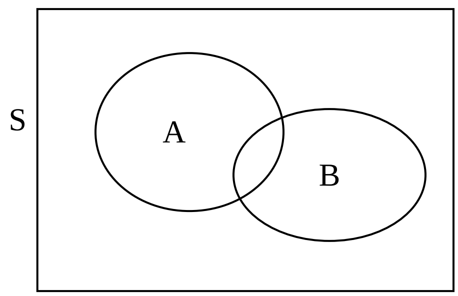
$A \cup B$={ PPP, PPF, FPP, PFP}=B, $A \cap B$={PPF, FPP, PFP}=A

- Sometimes it is useful to use Venn diagram to visualize the relationships between events

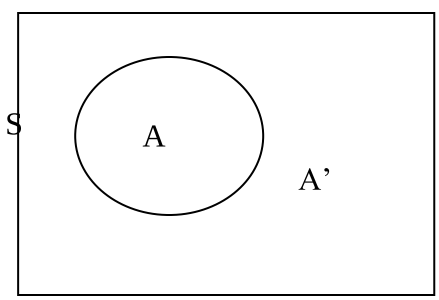
(1) $A \cup B$, the union of events A and B. It reads as "A ***union*** B" or "A ***or*** B" (The area covered by either A or B)



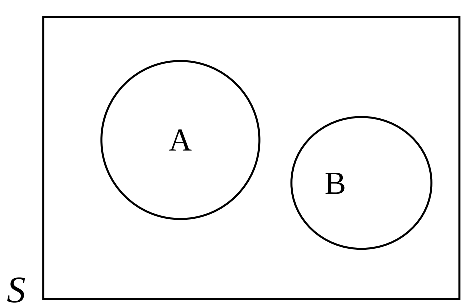
(2) $A \cap B$, the intersection of events A and B. It reads as "A ***intersect*** B" or "A ***and*** B" (The area covered by both A and B)



(3) *A'*, the complement of event A. It reads as "A ***complement***" or "***not*** A" (The area inside S but not covered by A)



(4) A and B are disjoint. That is, $A \cap B = \Phi$ (A and B do not have common part)



Probability

The probability of an event, *A*, denoted as P(*A*), is a quantity to describe how likely event *A* occurs.

Ex. P(*A*) = 0 $\Rightarrow$ Event A will never occur

Axiom of probability

1. The probability of any event must lie between <u>0</u> and <u>1</u>.

   That is, for any event A,

$$0 \le P(A) \le 1$$

2. The total probability assigned to the sample space of an experiment must be <u>1</u>.

   That is, P(S) = 1

Properties of Probability

1. **<u>The addition rule</u>**: for any 2 events *A* and *B*,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

   (this should be clear if we view P(A) is the area covered by A in the sample space S)

2. If *A* and *B* are **<u>disjoint</u>**, then $P(A \cap B) = 0$

   $\Rightarrow$ As a result, the addition rule *<u>for disjoint events</u>* can be simplified as

   $P(A \cup B) = P(A) + P(B)$ (only true if A and B are disjoint)

3. **<u>The complement rule</u>**: for any event *A*,

   P(*A'*) = 1 – P(*A*)

   Proof:

   $S = A \cup A'$, A and A' are disjoint. So

   1 = P(S) = P(A) + P(A')

Ex. A student is randomly selected from a class where 35% of the class is left-handed and 50% are sophomores. We further know that 5% of the class consists of left-handed sophomores.

(1) What is the probability of selecting a student is either left handed OR a sophomore?

- What we know:

  Define A = event that a randomly selected student is left-handed

  B = event that a randomly selected student is a sophomore

  P(A) = 0.35, P(B) = 0.5, and $P(A\cap B) = 0.05$

- What we want: $P(A\cup B)$

- Solve: $P(A\cup B) = P(A)+P(B)-P(A\cap B) = 0.35+05-0.05 = 0.8$

(2) What is the probability of selecting a right-handed sophomore?

- What we want: $P(A'\cap B)$

- Solve: We can view from the Venn diagram that $B = (A'\cap B)\cup(A\cap B)$. So

$P(B) = P(A'\cap B)+P(A\cap B)$ (since $(A'\cap B)$ *and* $(A\cap B)$ are disjoint). That is,

$$P(A'\cap B) = P(B)-P(A\cap B) = 0.5-0.05 = 0.45$$

(3) Are the events of *selecting a left-handed student* and *selecting a sophomore* considered to be disjoint? Why?

- What we want: Are A and B disjoint? That is, is $A\cap B=\phi$?

- Solve: If $A\cap B=\phi$, then $P(A\cap B)=P(\phi)=0.$ But it is given that $P(A\cap B)=0.05$>0, so A and B cannot be disjoint.

Ex. A certain system can experience 2 different types of defects. Let A_i , *i*=1,2, denote the event that the system has a defect of type *i*. Suppose that

$$P(A_1) = .15, P(A_2) = .10, P(A_1 \cup A_2) = 0.17$$

(1) What is the probability that the system has both type 1 and type 2 defects?

- What we know:

$$P(A_1) = .15, P(A_2) = .10, P(A_1 \cup A_2) = 0.17$$

- What we want: $P(A_1 \cap A_2)$

- Solve: Since $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$, so

$$P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.15 + 0.1 - 0.17 = 0.08$$

(2) What is the probability that the system has at least one type of defects?

- What we want: $P(A_1 \cup A_2)$

- Solve: It is given to be 0.17

(3) What is the probability that the system has no defects?

- What we want: $P[(A_1 \cup A_2)']$

- Solve: $P[(A_1 \cup A_2)'] = 1 - P(A_1 \cup A_2) = 1 - 0.17 = 0.83$