Connecting Algebra and Geometry Through Coordinates **Name: _________________**

Date: ________________

MCC9-12.G.GPE.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
MCC9-12.G.GPE.5 Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
MCC9-12.G.GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
MCC9-12.G.GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MCC9-12.G.CO.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

The focus of this unit is to have students analyze and prove geometric properties by applying algebraic concepts and skills on a coordinate plane. Students learn how to prove the fundamental theorems involving parallel and perpendicular lines and their slopes, applying both geometric and algebraic properties in these proofs. They also learn how to prove other theorems, applying to figures with specified numerical coordinates. (A theorem is any statement that is proved or can be proved. Theorems can be contrasted with postulates, which are statements that are accepted without proof.)

Lesson 6.1 Distance Formula

DISTANCE FORMULA: The distance between points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1 **Find the distance between** $(3, -2)$ **and** $(-2, 4)$.

Solution Let $(x_1, y_1) = (3, -2)$ and $(x_2, y_2) = (-2, 4)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-2-3)^2 + (4-(-2))^2} = \sqrt{(-5)^2 + (6)^2} = \sqrt{61}$$

PROBLEMS

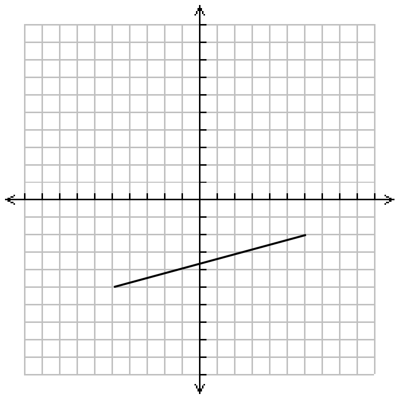
Find the distance between the two points.

1. $(5, 2), (3, 8)$

2. $(-2, 0), (-4, 5)$

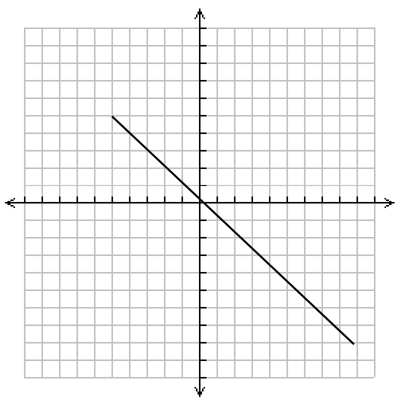
3. $(7, -1), (-5, 3)$

4. Find the length of the line shown (Hint: find the end points, then use the distance formula).



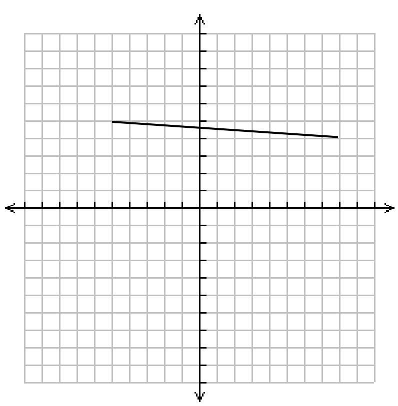
Length:

5. Find the length of the line shown (Hint: find the end points, then use the distance formula).



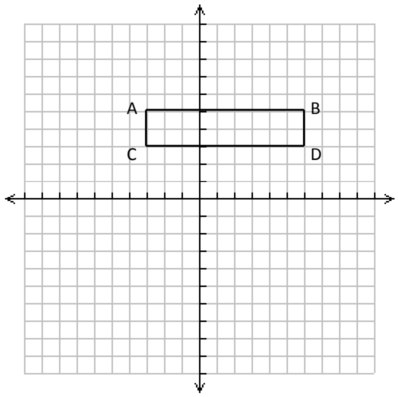
Length:

6. Find the length of the line shown (Hint: find the end points, then use the distance formula).



Length:

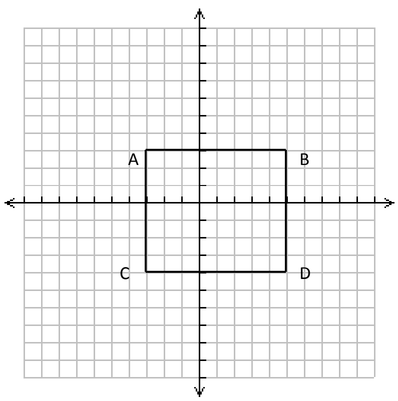
Example 3 Find the perimeter and area of the rectangle ABCD. Each unit represents 1 cm.



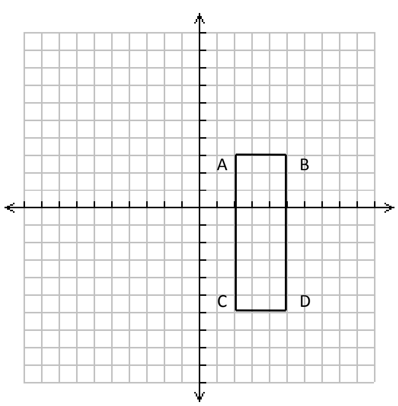
Solution:
To find the perimeter just add all of the sides: 2+9+2+9=22cm.
To find the area multiply base times height: 9 x 2 = 18 cm^2.

PROBLEMS

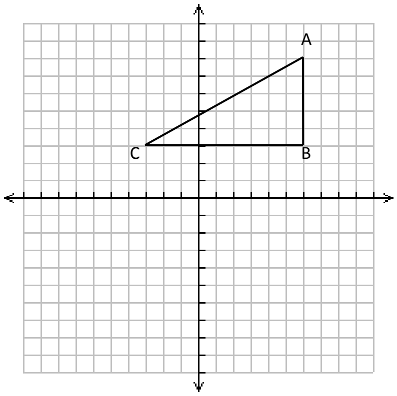
7. Find the perimeter and area of the rectangle ABCD. Each unit represents 1 cm.



8. Find the perimeter and area of the rectangle ABCD. Each unit represents 1 cm.



Example 4 Find the perimeter and area of the triangle ABC. Each unit represents 1 cm.



Solution:
To find the perimeter just add all of the sides
AB=5 and BC=9. AC is harder to find: Use the Pythagorean theorem to solve for AC:

$$AC^2 = AB^2 + BC^2$$
$$AC^2 = 5^2 + 9^2 = 106$$
$$\rightarrow AC = \sqrt{106} \approx 10.3cm$$

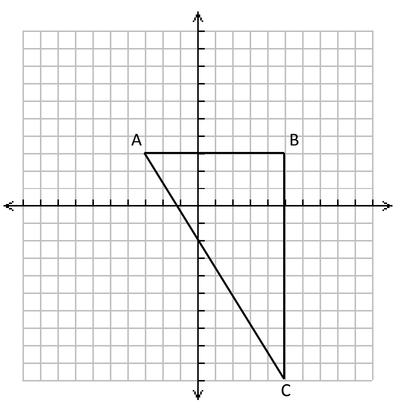
Therefore, the perimeter is 5 + 9 + 10.3 = 24.3 cm.

To find the area of a triangle use the formula: $A = \frac{1}{2}bh$ where $b = base$ and $h = height$.

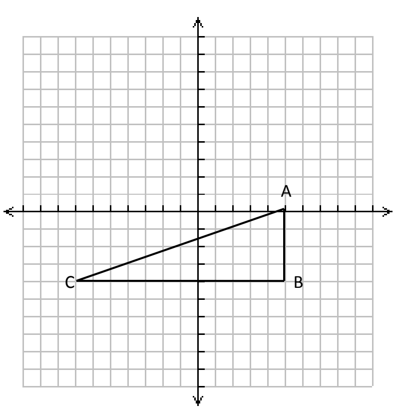
Let $AB = base = b = 5$ and $BC = height = h = 9$, then $A = \frac{1}{2}bh = \frac{1}{2}5 \cdot 9 = 22.5cm^2$. **The area is 22.5cm².**

PROBLEMS

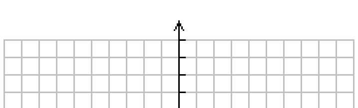
9. Find the perimeter and area of the triangle ABC. Each unit represents 1 cm.

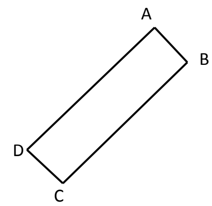


10. Find the perimeter and area of the triangle ABC. Each unit represents 1 cm.

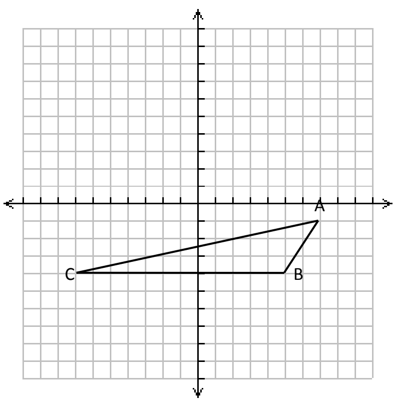


11. Find the perimeter and area of the rectangle ABCD. Each unit represents 1 cm.

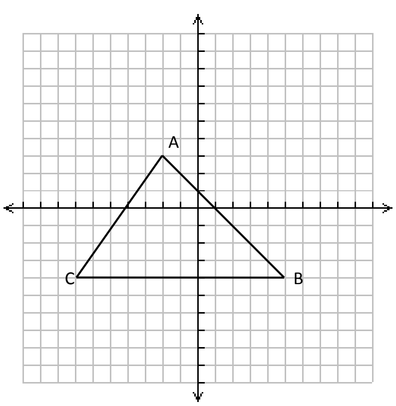




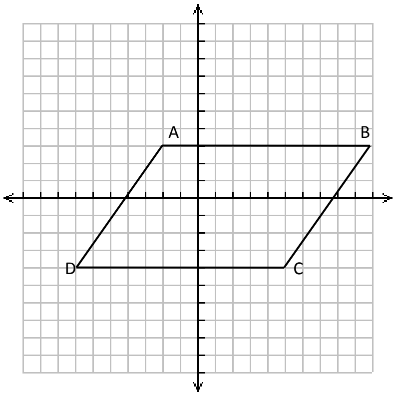
12. Find the perimeter of the triangle ABC. Each unit represents 1 cm.



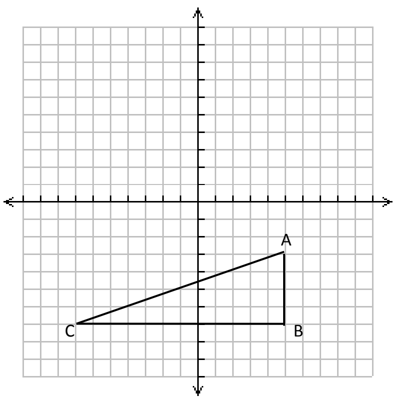
13. Find the perimeter of the triangle ABC. Each unit represents 1 cm.



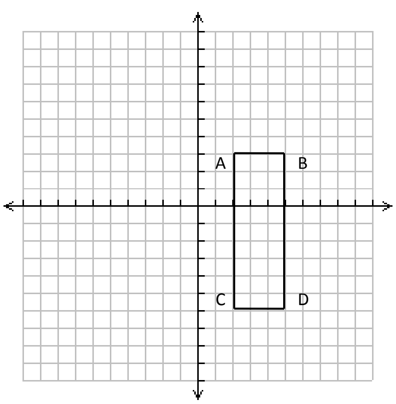
14. Find the perimeter of the parallelogram ABCD. Each unit represents 1 cm.



15. How can you double the area of the triangle ABC? Each unit represents 1 cm.



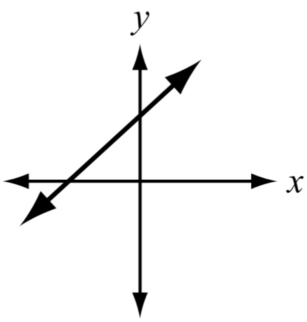
16. How can you double the area of the rectangle ABCD? Each unit represents 1 cm.



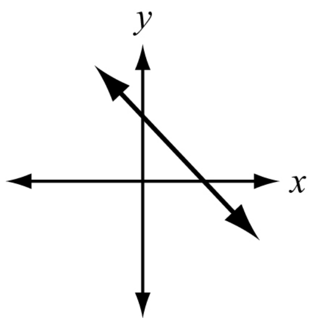
Lesson 6.2 Slope Formula

SLOPE FORMULA: The slope of the line through points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$.

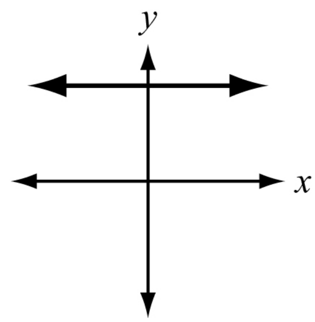
Slopes can be positive, negative, 0, or undefined.



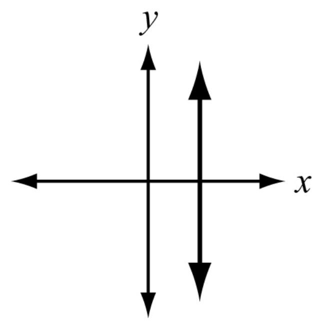
A line with a positive slope slants up to the right.



A line with a negative slope slants down to the right.

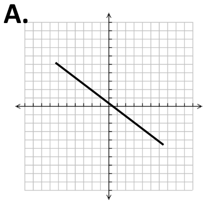


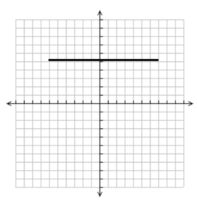
A line with a slope of 0 is horizontal.

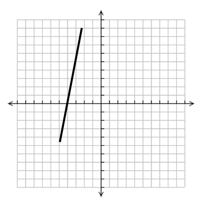


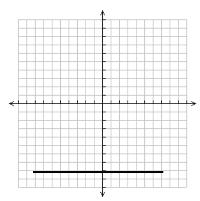
A line with an undefined slope is vertical.

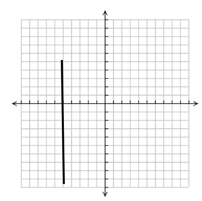
PROBLEMS:
Is the slope positive, negative, 0, or undefined? Circle the correct answer.

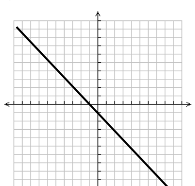
A.

Pos Neg Zero Undefined

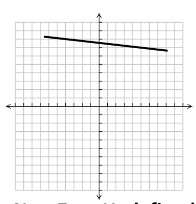
B.

Pos Neg Zero Undefined

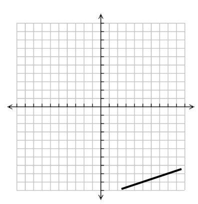
C.

Pos Neg Zero Undefined

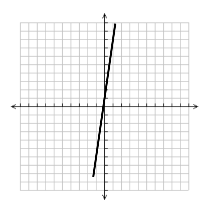
D.

Pos Neg Zero Undefined

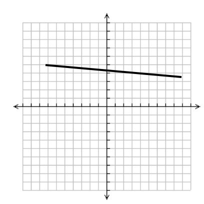
E

Pos Neg Zero Undefined

F.

Pos Neg Zero Undefined

G.

Pos Neg Zero Undefined

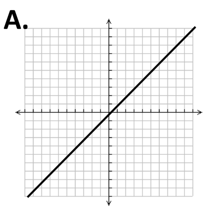
H.

Pos Neg Zero Undefined

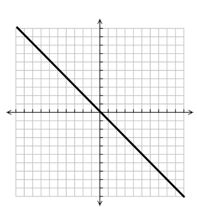
I.

Pos Neg Zero Undefined

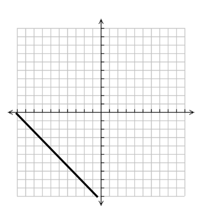
J.

Pos Neg Zero Undefined

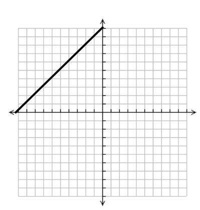
Lines and their slopes are related by the following properties:

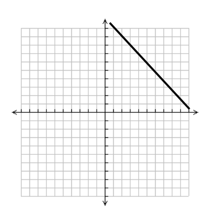
a. Two nonvertical lines are **parallel** if and only if they have equal slopes.
b. Two nonvertical lines are **perpendicular** if and only if the product of their slopes is –1.
PROBLEMS:
What is the slope of each line? Which lines are parallel and which ones are perpendicular?

A.

Slope:_______

B.

Slope:_______

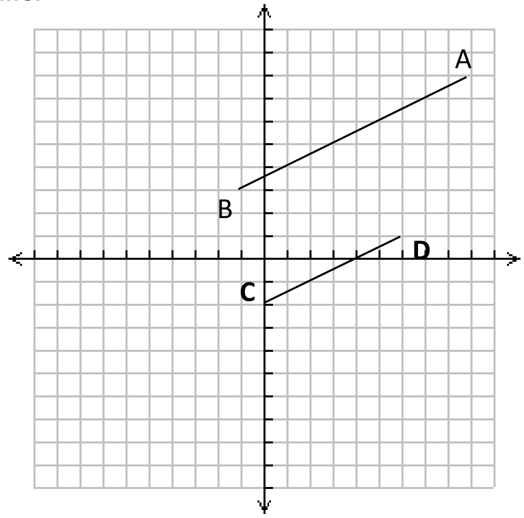
C.

Slope:_______

D.

Slope:_______

E.

Slope:_______

Example 1 **Use the slope formula to prove that lines are parallel**

Show that AB has the same slope as CD.



Solution: First find the coordinates for each point:

$A(9,8), B(-1,3), C(0,-2), D(6,1).$

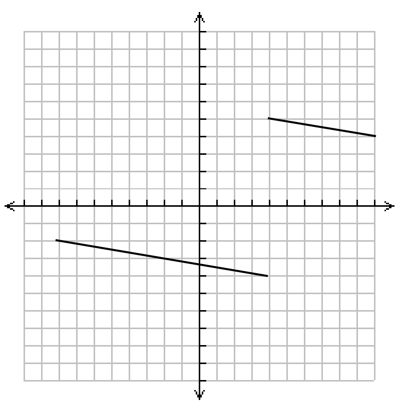
Then find the slope for each line:

Slope of $AB = \frac{3-8}{-1-9} = \frac{-5}{-10} = \frac{1}{2}$ Slope of $CD = \frac{1--2}{6-0} = \frac{3}{6} = \frac{1}{2}$

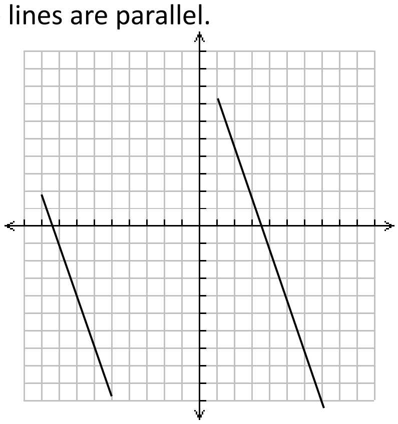
Since AB and CD have the same slope they are parallel.

PROBLEMS

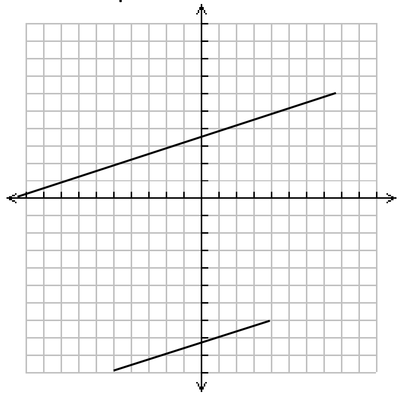
1. Show that the lines are parallel.



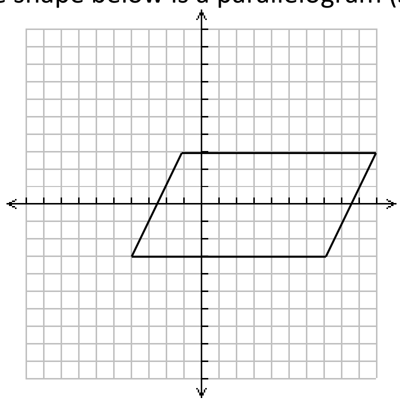
2. Show that the lines are parallel.



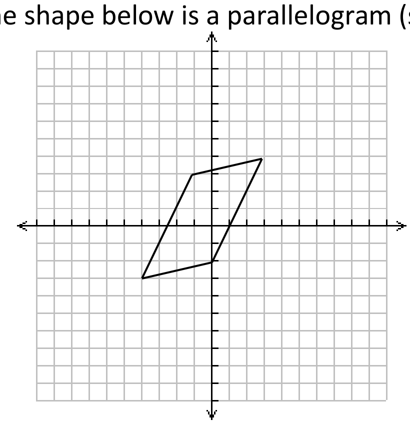
3. Show that the lines are parallel.



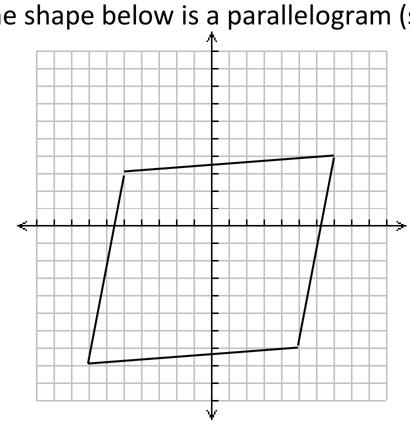
4. Prove that the shape below is a parallelogram (show that the opposing sides have the same slope).



5. Prove that the shape below is a parallelogram (show that the opposing sides have the same slope).

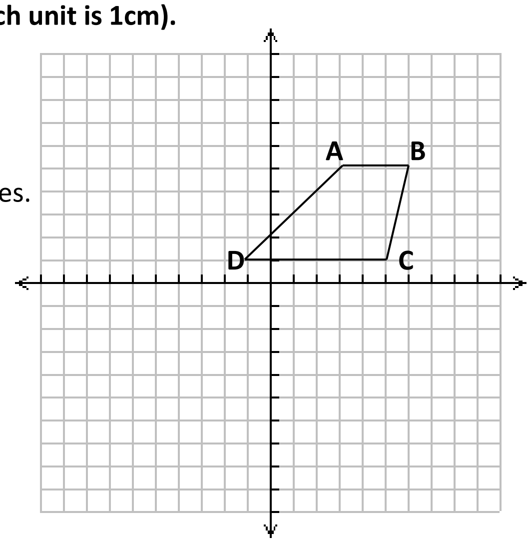


6. Prove that the shape below is a parallelogram (show that the opposing sides have the same slope).



Lesson 6.3 Perimeter and Area of Trapezoids

Example 1 **Find the perimeter and area of the trapezoid (each unit is 1cm).**



Solution: To find the perimeter add up all of the sides.

$$AB = 3, CD = 6$$
$$BC = \sqrt{4^2 + 1^2} = \sqrt{17} \approx 4.12$$
$$DA = \sqrt{4^2 + 4^2} = \sqrt{32} \approx 5.66$$

Adding up all the sides (3+6+4.12+5.66) yields 18.78.
Therefore, the perimeter is 18.78cm.

To find the area use the formula $Area = \frac{1}{2}(b_1 + b_2) \cdot h$ where

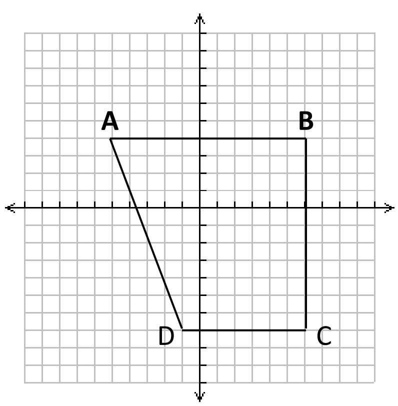
$h = height$ $b_1 = top\,base$ $b_2 = bottom\,base$

$$Area = \frac{1}{2}(3+6) \cdot 4 = 18cm^2$$

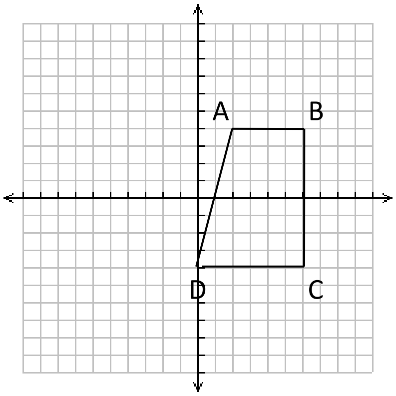
Therefore, the area is $18cm^2$.

PROBLEMS

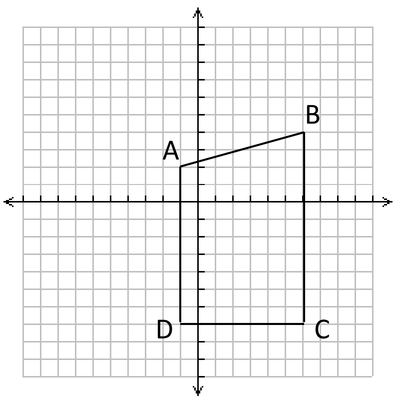
1. Find the perimeter and area of the trapezoid (each unit is 1cm).



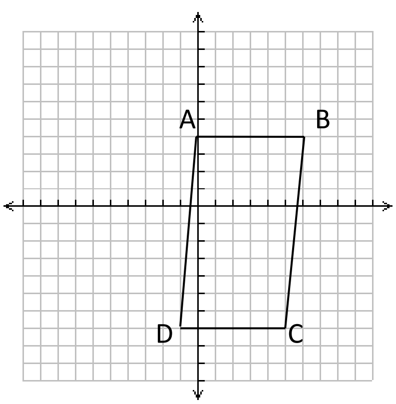
2. Find the perimeter and area of the trapezoid (each unit is 1cm).



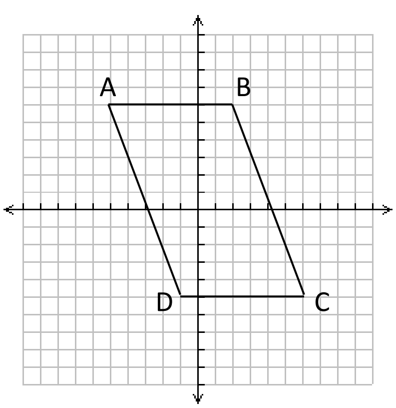
3. Find the perimeter and area of the trapezoid (each unit is 1cm).



4. Find the perimeter and area of the parallelogram (each unit is 1cm).



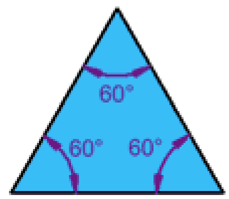
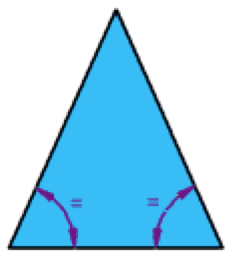
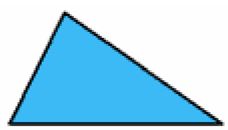
5. Find the perimeter and area of the parallelogram (each unit is 1cm).



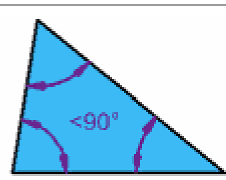
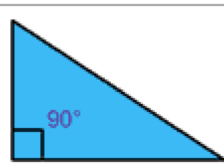
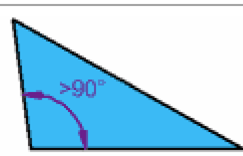
Lesson 6.4 Classifying Triangles

A triangle has three sides and three angles
The three angles always add to 180°

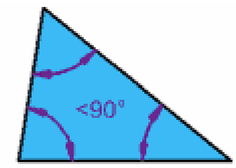
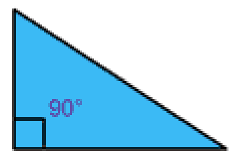
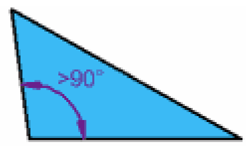
There can be 3, 2 or no equal sides/angles:

	**Equilateral Triangle** **Three** equal sides **Three** equal angles, always 60°
	**Isosceles Triangle** **Two** equal sides **Two** equal angles
	**Scalene Triangle** **No** equal sides **No** equal angles

Triangles can also have names that tell you what **type of angle** is inside*:*

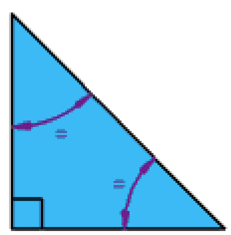
	**Acute Triangle** All angles are less than 90°
	**Right Triangle** Has a right angle (90°)
	**Obtuse Triangle** Has an angle more than 90°

Triangles can also have names that tell you what **type of angle** is inside*:*

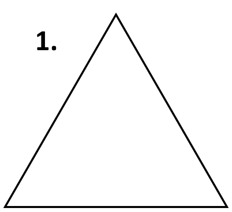
	**Acute Triangle** All angles are less than 90°
	**Right Triangle** Has a right angle (90°)
	**Obtuse Triangle** Has an angle more than 90°

Combining the Names

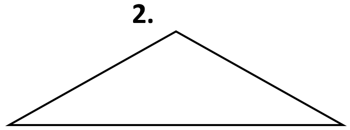
Sometimes a triangle will have two names, for example:

	**Right Isosceles Triangle** Has a right angle (90°), and also two equal angles Can you guess what the equal angles are? Answer: Each one is 45°.

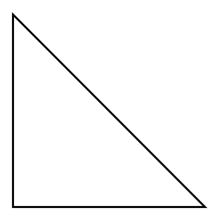
PROBLEMS: **Classify the triangles.**

1. 

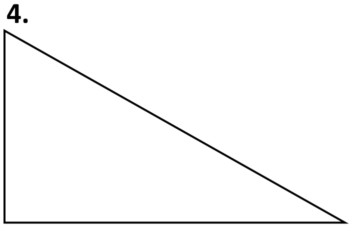
____

2. 

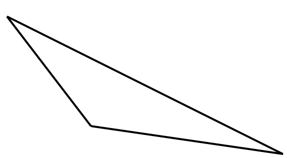
____

3. 

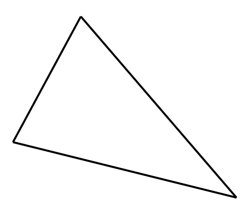
____

4. 

____

5. 

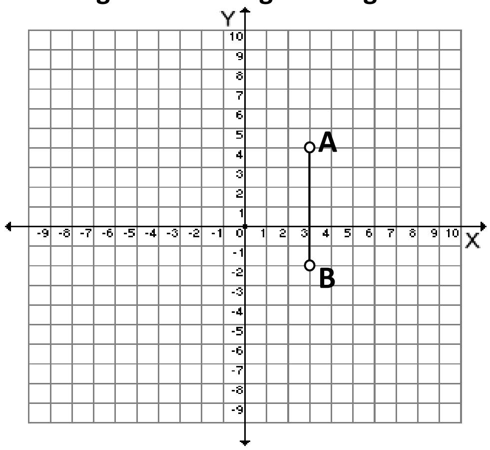
____

6. 

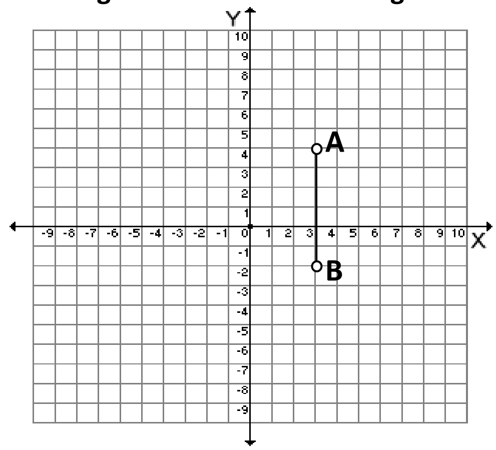
____

7. Determine the location of point C such that triangle ABC has each given characteristics.

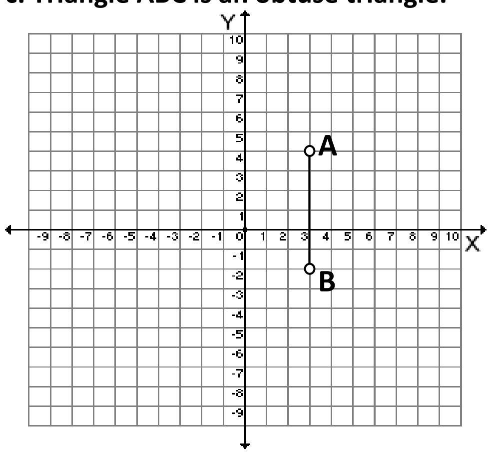
a. Triangle ABC is a right triangle.



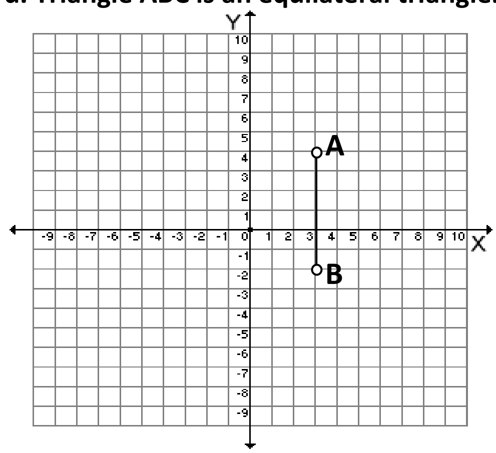
b. Triangle ABC is an acute triangle.



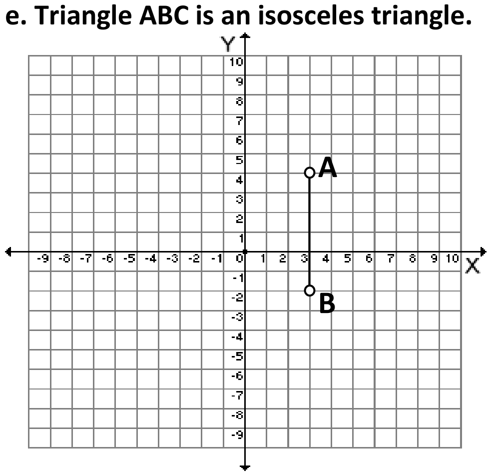
c. Triangle ABC is an obtuse triangle.



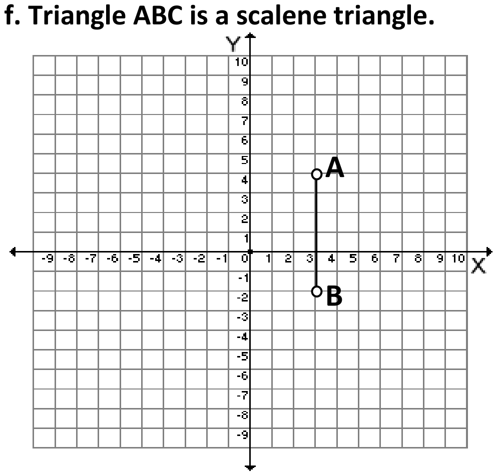
d. Triangle ABC is an equilateral triangle.



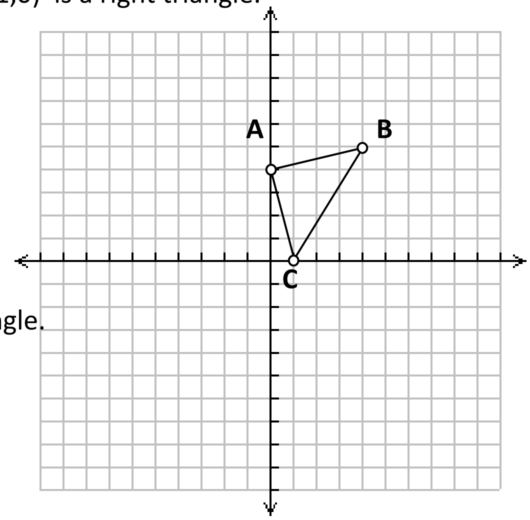
e. Triangle ABC is an isosceles triangle.



f. Triangle ABC is a scalene triangle.



Example 1 Determine whether triangle ABC A(0,4), B(4,5), C(1,0) is a right triangle.



Solution: Draw the triangle.

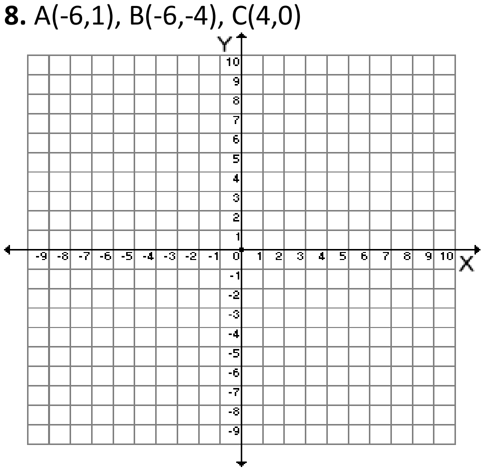
Then find the slope AB and slope AC. If the slopes are negative inverse to each other, then the lines are perpendicular to each other and form a 90˚ angle.

$$Slope\ AB = \frac{1}{4} \qquad Slope\ AC = \frac{4}{-1} = -4$$

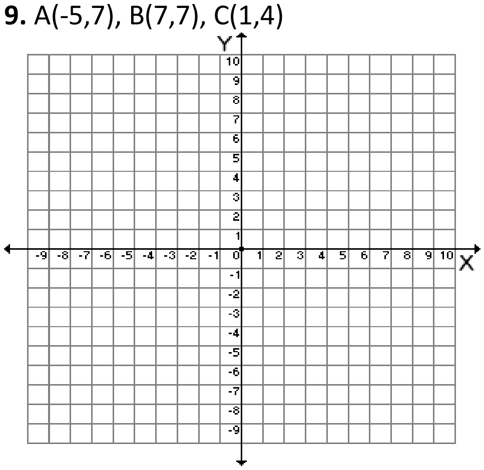
Since ¼ and -4 are negative inverses to each other, AB and AC are perpendicular and form a 90˚ angle. Thus triangle ABC is a right triangle.

PROBLEMS Determine whether triangle ABC is a right triangle. If it is not a right triangle, is it acute or obtuse?

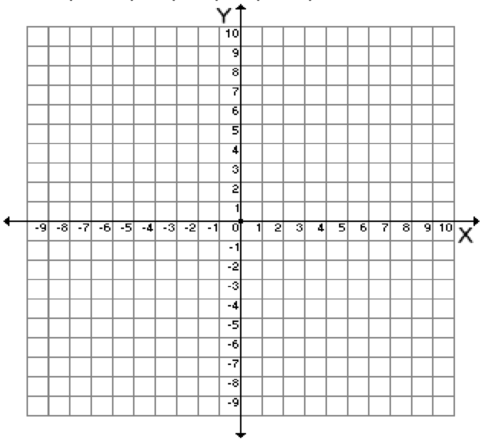
8. A(-6,1), B(-6,-4), C(4,0)



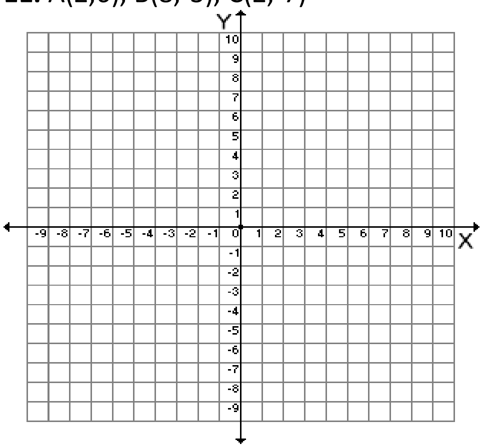
9. A(-5,7), B(7,7), C(1,4)



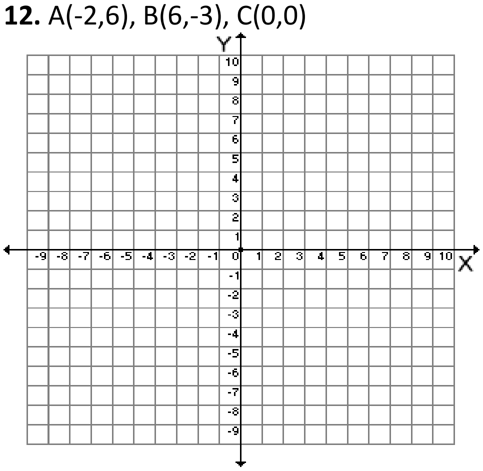
10. A(-4,-1), B(1,3), C(3,-4)



11. A(2,6), B(8,-3), C(2,-7)

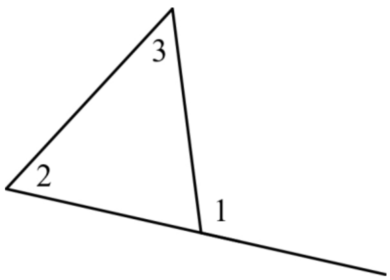


12. A(-2,6), B(6,-3), C(0,0)



Exterior Angle Theorem

The measure of each exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.



$m\angle 1 = m\angle 2 + m\angle 3$

Example: Look at the picture above. If angle 2 = 40˚ and angle 3 = 30˚ what is the measure for angle 1?
Solution: $m\angle 1 = m\angle 2 + m\angle 3 = 40^\circ + 30^\circ = 70^\circ$.

PROBLEMS

13. Look at the picture above. If angle 2 = 35˚ and angle 3 = 80˚ what is the measure for angle 1?

14. Look at the picture above. If angle 2 = 90˚ and angle 3 = 45˚ what is the measure for angle 1?

15. Look at the picture above. If **angle 1** = 67˚ and angle 3 = 45˚ what is the measure for angle 2?

16. Look at the picture above. If **angle 1** = 51˚ and angle 2 = 30˚ what is the measure for angle 3?

Some useful properties of proportions state that all of the following are equivalent:

if $\frac{a}{b} = \frac{c}{d}$ **then** $ad = bc$ **and** $\frac{a}{c} = \frac{b}{d}$

Example: If a=5, b=3, c=10, and d=6 then

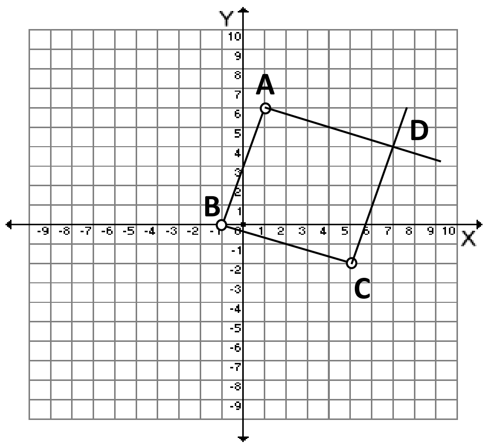
$\frac{5}{3} = \frac{10}{6}$ **and** $5 \cdot 6 = 10 \cdot 3$ **and** $\frac{5}{10} = \frac{3}{6}$

18. Solve for x: $\frac{x}{7} = \frac{14}{28}$ **19.** Solve for x: $\frac{4}{x} = \frac{28}{40}$ **20.** Solve for x: $\frac{45}{75} = \frac{5}{x}$

Example:

For the set of given points, determine the location of the fourth point such that the described figure is created.

Determine the location of vertex D such that the quadrilateral is a square. A(1,6), B(-1,0), C(5,-2).



Find the slope of AB and BC: $Slope\ AB = \frac{6}{2} = 3$ $\quad Slope\ BC = \frac{-2}{6} = -\frac{1}{3}$

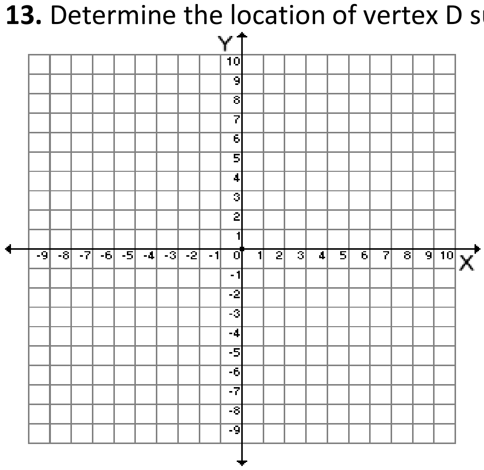
Draw a line through point A with a slope of $-\frac{1}{3}$ and through point C with a slope of 3.

Both lines meet at point D (7,4). Therefore**, the coordinates of vertex D such that the quadrilateral is a square is (7,4).**

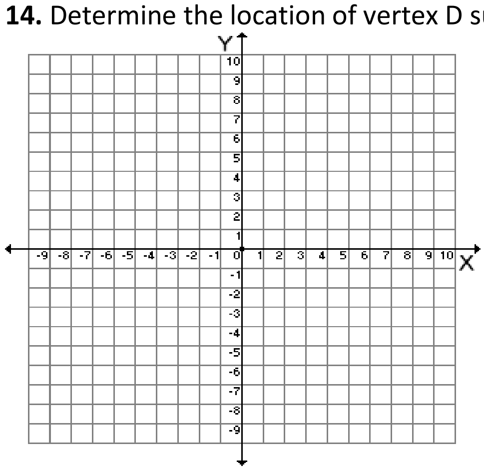
PROBLEMS

For the set of given points, determine the location of the fourth point such that the described figure is created.

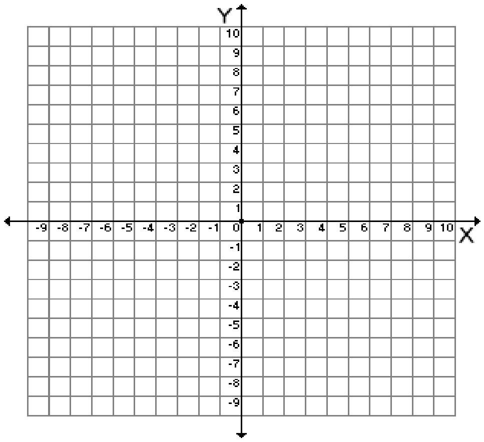
13. Determine the location of vertex D such that the quadrilateral is a square. A(2,4), B(8,0), C(4,-6)



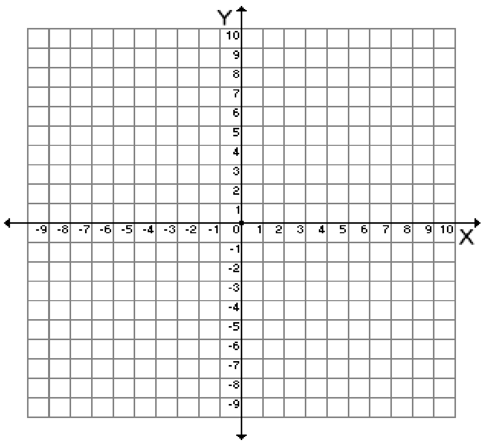
14. Determine the location of vertex D such that the quadrilateral is a trapezoid. A(-3,5), B(-5,1), C(-1,1)(Multiple solutions)



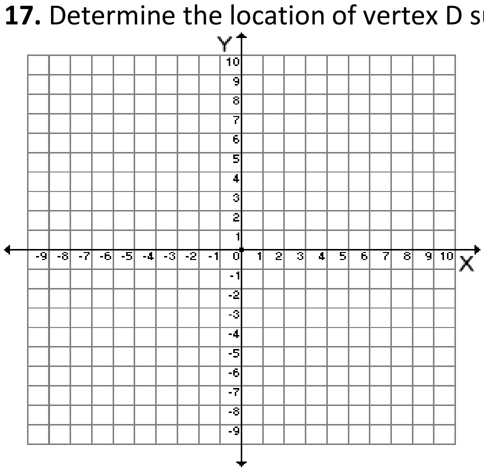
15. Determine the location of vertex D such that the quadrilateral is a square. A(-2,-2), B(0,6), C(8,4)



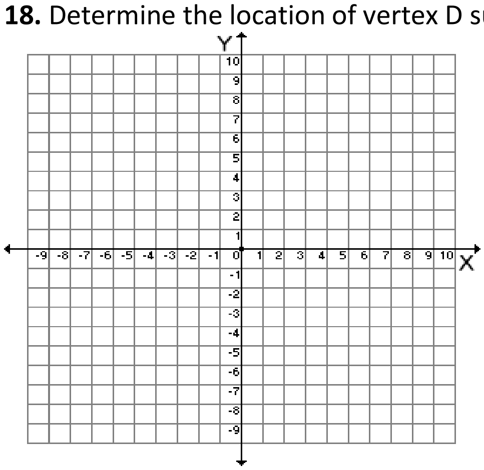
16. Determine the location of vertex D such that the quadrilateral is a trapezoid. A(6,2), B(2,-4), C(-4,0). (Multiple solutions)



17. Determine the location of vertex D such that the quadrilateral is a square. A(0,0), B(5,0), C(5,5).



18. Determine the location of vertex D such that the quadrilateral is a trapezoid. A(-4,-1), B(0,0), C(1,5). (Multiple solutions)



Lesson 6.5 Circles in the Coordinate Plane

Example Assume circle A has a center at the origin and a radius 13. Determine if point B (5,12) lies on the circle.

Solution: Determine the distance of point B from the center of the circle at the origin (0,0).

Use the distance formula between points (5,12) and (0,0):

$$d=\sqrt{(y_2-y_1)^2+(x_2-x_1)^2}=\sqrt{(12-0)^2+(5-0)^2}=13$$

Since the distance from the origin to point (5,12) is the same as the radius (13), the point lies on the circle.

(If the radius and the distance are not the same then the point does not lie on the circle.)

PROBLEMS

For each questions assume circle A has a center at the origin and a given radius or diameter. Determine if each point B lies on the circle.

1. Radius: 6; B(4,5)

2. Radius: 10; B(6,8)

3. Radius: 5; B(-3,4)

4. Radius: 2.5 ; B(-1.5,-2.5)

5. Radius: 5 ; B(0,5)

Example

Assume circle A has a center at (6,5), a radius of 15. Determine if point C (3,9) on the circle.

Solution: Determine the distance between point C (3,9)and the center of the circle at (6,5). If the distance between the center and the point is the same as the radius then the point has to lay on the circle.

Use the distance formula: $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(9-5)^2 + (3-6)^2} = \sqrt{25} = 5$

Since the distance between the origin and the point is the same as the radius (5), the point lies on the circle.

(If the radius and the distance are not the same then the point does not lie on the circle.)

PROBLEMS

For each questions assume circle A has a center and radius as given. Determine if point C lies on the circle.

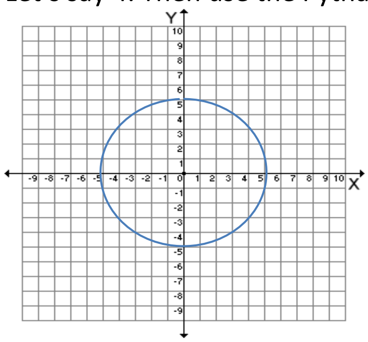
6. Center: (-8,-7); Radius: 15; C(1,4)

7. Center: (8,2); Radius: 2; C(7,-1)

8. Center: (1,0); Radius: 5; C(1,5)

Example Assume circle A has a center at the origin and has a radius of 5. Complete the table by determining the coordinates of the five points described.

Solution:
In order to find a point in quadrant 1, pick a number for x which is less than 5 (the radius of the circle). Let's say 4. Then use the Pythagorean theorem to solve for the corresponding y-value:



$$a^2 + b^2 = c^2$$
$$4^2 + b^2 = 5^2$$
$$16 + b^2 = 25$$
$$b^2 = 9$$
$$b = \pm 3$$

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
5	(5,0)	(0,5)	(4,3)
	(-5,0)	(0,-5)	

PROBLEMS
Assume circle A has a center at the origin and has a radius as given. Complete the table by determining the coordinates of the five points described.

9. Radius: 15

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
15			

10. Radius: 13

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
13			

11. Radius: 2.5

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
2.5			

12. Radius: 7.5

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
7.5			

13. Radius:6.5

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
6.5			

14. Radius: 17

Radius of circle A	x-intercepts	y-intercepts	Point in Quadrant 1
17			

Lesson 6.6 Inductive Reasoning

Goal: Describe patterns and use inductive reasoning.

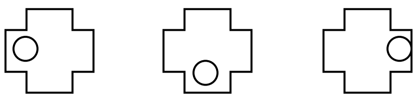
Vocabulary:

A **conjecture** is an unproven statement that is based on observations.

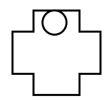
You use **inductive reasoning** when you find a pattern in specific cases and then write the conjecture for the general case.

A **counterexample** is a specific case for which a conjecture is false.

Example 1 Describe how to sketch the next figure in the pattern. Then sketch the next figure.



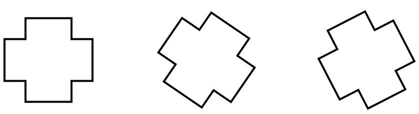
Solution: Each figure looks like the one before except that it has been rotated 90˚counterclockwise. Sketch the next figure by rotating it another 90˚ counterclockwise.



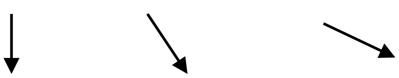
PROBLEMS:

Describe how to sketch the next figure in the pattern. Then sketch the next figure.

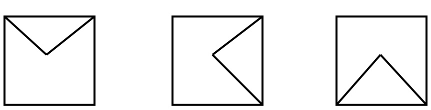
1.



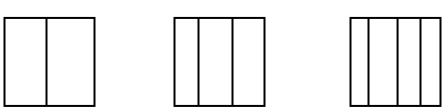
2.



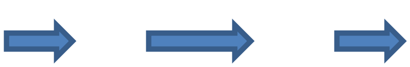
3.



4.



5.



Example 2 Describe the pattern in the numbers 2, 8, 32, 128, … and write the next three numbers in the pattern.

Solution: Each number in the pattern is four times the previous number. 2 x 4 = 8, 8 x 4 = 32, 32 x 4 = 128, and so on. Continue the pattern for the next three numbers:

128 x 4 = 512, 512 x 4 = 2048, 2048 x 4 = 8192.

PROBLEMS:

Describe the pattern in the numbers. Then give the next numbers in the pattern.

5. 1, 5, 9, 13, …

6. 1, 3, 9, 27, 81, ..

7. -3, 0, 3, 6

8. 4, 6, 9, 13, 18, ..

9. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, ..$

10. 2, 3, 5, 8, 12, **..**

11. 1, 1, 2, 3, 5, 8, 13, 21,…..

12. 2, 4, 8, 16

Example 3 **Make and test a conjecture about the product of any two odd integers.**

Solution: Take several pairs of two odd integers and multiply them:

3 x 11 = 33 5 x 7 = 35 9 x 3 = 27 7 x 3 = 21

Conjecture: The product of any two integers is odd.

PROBLEMS:

13. Make and test a conjecture about the product about any two even integers**.**

14. Make and test a conjecture about the sum of an even integer and an odd integer.

Example 4 **Find a counterexample to show that the conjecture is false.**

Conjecture: All odd numbers are prime.

Solution: To find a counterexample, you need to find an odd number that is a composite number.

Because a counterexample exists, the conjecture is false.

PROBLEMS:

15. Find a counterexample to show that the conjecture is false. Conjecture: The difference of two positive numbers is always positive.

16. Find a counterexample to show that the conjecture is false. Conjecture: The quotient of two whole numbers is always a whole number.

17. Find a counterexample to show that the conjecture is false. Conjecture: A product of two numbers is always even.

Lesson 6.7 Analyze Conditional Statements

Goal: Write definitions as conditional statements.

Vocabulary: A **conditional statement** is a logical statement that has two parts, a hypothesis and a conclusion.

When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**.

The **negation** of a statement is the opposite of the original statement.

To write the **converse** of a conditional statement, just switch the hypothesis and the conclusion.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion.

To write the **contrapositive** of a conditional statement, first write the converse and then negate both the hypothesis and the conclusion.

If two lines intersect to from a right angle, then they are **perpendicular lines**.

Example 1 **Rewrite the conditional statement in if-then form**

a. All teachers have a college degree.

b. An angle is an acute angle if its measure is less than 90˚.

c. When $n=3$, then $n^2=9$.

Solution:

a. If you are a teacher, then you have a college degree.

b. If the measure of an angle is less than 90˚, then it is an acute angle.

c. If $n=3$, then $n^2=9$.

PROBLEMS: Rewrite the conditional statement in if-then form

1. On Sunday Jose has a soccer game.

2. The measure of a right angle is 90˚.

3. All sharks have a boneless skeleton.

Example 2	Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Soccer players are athletes." Decide whether each statement is true or false. **Solution:** **If-then form**: If you are a soccer player, then you are an athlete. (True, since soccer players are athletes) **Converse**: If you are an athlete, then you are a soccer player. (False, not all athletes are soccer players. **Inverse**: If you are not a soccer player, then you are not an athlete. (False, even if you don't play soccer, you still can be an athlete.) **Contrapositive**: If you are not an athlete, then you are not a soccer player. (True, a person who is not an athlete cannot be a soccer player).

PROBLEMS: **Write the if-then form, the converse, the inverse, and the contrapositive of the statement "Soccer players are athletes." Decide whether each statement is true or false.**

4. All 90˚ angles are right angles.

If-then form:

Converse:

Inverse:

Contrapositive:

5. All dogs are mammals.

If-then form:

Converse:

Inverse:

Contrapositive:

Lesson 6.8 Apply Deductive Reasoning

Vocabulary:

Deductive reasoning uses facts, definitions, and the laws of logic to form a logical argument. Deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen.

Inductive reasoning uses observed specific examples of a phenomenon to arrive at a general rule. The conclusion may not be necessarily true. Inductive reasoning looks at evidence and creates a general rule based on the evidence.

Law of Detachment: If the hypothesis of a true conditional statement is true then the conclusion is also true.

Law of Syllogism: If a then b (true statement)
If b then c (true statement)
Then the following statement is also true: If a then c.

Example 1 Use the Law of Detachment to make a valid conclusion

If the angle A is acute, then $0^\circ < \text{angle } A < 90^\circ$. Angle B is an acute angle. Using the law of detachment, what conclusion can you make?

Solution:

First identify the hypothesis (If the angle A is acute) and the conclusion (then $0^\circ < \text{angle } A < 90^\circ$).

Since angle B is acute, you can make the conclusion that $0^\circ < \text{angle } B < 90^\circ$.

PROBLEMS: Use the Law of Detachment to make a valid conclusion

1. If two angles have the same measure then they are congruent. You know that the measure of angle A equals the measure of angle B. Using the law of detachment, what conclusion can you make?

2. If B is between A and C, then AB + BC = AC. E is between D and F. Using the law of detachment, what conclusion can you make?

Example 2 **Use the Law of Syllogism**

Use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

If the electric power is off, then the refrigerator does not run. If the refrigerator does not run, then the food will spoil.

Solution:

The conclusion of the first statement is the hypothesis of the second statement. Therefore, you can write the following statement: If the electric power is off, then the food will spoil.

PROBLEMS: Use the Law of Syllogism to write the conditional statement that follows from the pair of true statements.

3. If you study hard, then you will pass all your classes. If you pass all your classes, then you will graduate.

4. If you work out, then you will gain muscle mass. If you gain muscle mass, then you will get stronger.

5. If it is night, then it is dark. If it is dark, I have problems seeing.

6. If I do my math homework, then I understand math better. If I understand math better, then I improve my math skills.

7. If I practice math, then I improve my problem solving skills. If I improve my problem solving skills, then I get smarter.

8. If I forget my homework, then I get written up. If I get written up, then I lose a day.

Example 3	**Use inductive reasoning**
	What conclusion can you make about the sum of two even integers?
	Solution:
	Using inductive reasoning:
	Look for a pattern in several examples: Then use inductive reasoning to make a conjecture.
	2+4=6, 6+8=14, -8+6=-2, 10+20=30
	Conjecture: even integer + even integer = even integer

PROBLEMS: Use inductive reasoning

9. What conclusion can you make about the sum of two odd integers?

10. What conclusion can you make about the product of an even integer and an odd integer?

11. What conclusion can you make about the difference of two even integers?

Example 3 **Use deductive reasoning**

What conclusion can you make about the sum of two even integers?

Solution:

Using deductive reasoning:

Let n and m be any integer. 2n and 2m are even integers because any integer multiplied by 2 is even.

2n + 2m represent the sum of two even integers.

2n + 2m can be rewritten as 2(n+m). Any integer multiplied by 2 is even, therefore, 2(n+m) is even. You can conclude that the sum of two even integers is an even integer.

PROBLEMS: Use deductive reasoning

12. What conclusion can you make about the sum of two odd integers? (An odd integer can be written as 2n+1)

13. What conclusion can you make about the product of an even integer (2n) and an odd integer (2m+1)?

14. What conclusion can you make about the difference of two even integers (2n and 2m)?

GENERAL LOGICAL REASONING

15. Isabella sees 5 red fire trucks. She concludes that all fire trucks are red. Is her conclusion necessarily correct? Does she use inductive or deductive reasoning?

16. You read an article on the internet that says that a high-fat diet increases a person's risk of heart disease. You know that your father has a lot of fat in his diet. What conclusion can you draw? Do you use inductive or deductive reasoning?

17. Your teacher tells you that increased sun exposure without sunblock increases the risk of skin cancer. Your friend Sue spends as much time as possible in the sun in order to work on her tan. What conclusion can you draw? Do you use inductive or deductive reasoning?

18. Hannah tells you that she has been in the mall three times the past week, and every time there were a lots of people there. She says: "It is always crowded at the mall." Is her conclusion necessarily correct? Does she use inductive or deductive reasoning?

19. You earn $15/hour babysitting. Next week you are scheduled to babysit for 6 hours. What conclusion can you draw? Do you use inductive or deductive reasoning?